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# Peculiar Properties of the Financial State of Companies with Falling Income

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## ABSTRACT

The recent rise in inflation in Europe, caused by the pandemic, the increase in prices for energy resources and the violation of the logistics of energy supplies, has led to a decrease in company income. This makes it relevant to study the financial condition of companies with falling revenues. The **purpose** of this study is the development of tools for quantifying the impact of falling company incomes is becoming essential for making adequate management decisions. Until recently, such tools in capital structure theory did not exist. Two main theories of the capital cost and capital structure – Brusov – Filatova – Orekhova (BFO) theory and Modigliani – Miller (MM) theory – described companies with constant revenue: the first – for arbitrary age company, the second – for perpetuity companies. Within last couple years both these theories have been generalized for the case of variable revenue. In this paper the peculiar properties of the financial state of companies with falling income are studied within the modern capital cost and capital structure theory – Brusov–Filatova–Orekhova (BFO) theory, generalized for the case of variable revenue. As part of the goal, the **tasks** are solved to study the behavior of the main financial indicators (the cost of raising capital, the discount rate, the company's capitalization, the cost of equity, and others), their dependence on debt financing, the age of the company in the face of declining income, which will make it possible to make adequate management decisions and reduce risks for companies.

**Keywords:** Generalized Brusov–Filatova–Orekhova (BFO) theory; the companies with falling income; equity cost; the weighted average cost of capital; company capitalization

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## ОРИГИНАЛЬНАЯ СТАТЬЯ

# Особенности финансового состояния компаний с убывающими доходами

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## АННОТАЦИЯ

Недавний рост инфляции в Европе, вызванный пандемией, ростом цен на энергоресурсы и нарушением логистики энергопоставок, привел к снижению доходов компаний. Это делает актуальным изучение финансового состояния компаний с падающей выручкой. **Целью** настоящего исследования является разработка инструментов для количественной оценки влияния падения доходов компании на ее финансовые индикаторы, что необходимо для принятия адекватных управленческих решений. До недавнего времени таких инструментов в теории структуры капитала не существовало. Две основные теории стоимости капитала и структуры капитала – Брусова–Филатовой–Ореховой (БФО) и Модильяни–Миллера (ММ) – описывали компании с постоянным доходом: первая – для компаний произвольного возраста, вторая – для бессрочных компаний. За последние пару лет обе эти теории были обобщены на случай переменных доходов. В данной работе особенности финансового состояния компаний с падающими доходами исследуются в рамках современной теории стоимости и структуры капитала – теории Брусова–Филатовой–Ореховой (БФО), обобщенной на случай переменных доходов. В рамках поставленной цели решаются **задачи** по изучению поведения основных финансовых показателей (стоимости привлечения капитала, ставки дисконтирования, капитализации компании, стоимости собственного капитала и других), их зависимости от заемного финансирования, возраста компании в условиях снижения доходов, что позволит принять адекватные управленческие решения и снизить риски для компаний.

**Ключевые слова:** обобщенная теория Брусова–Филатовой–Ореховой (БФО); компании с убывающими доходами; стоимость собственного капитала; средневзвешенная стоимость капитала; капитализация компании

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## INTRODUCTION

The capital structure affects all of a company's financial performance, including capitalization and the cost of raising capital. Until recently, two main theories of the capital cost and capital structure – Brusov–Filatova–Orekhova (BFO) theory and Modigliani – Miller (MM) theory – described companies with constant income: the first – for arbitrary age company, the second – for perpetuity companies. Within last couple years, both these theories have been generalized to the case of variable revenue. This allows study the financial state of companies with falling revenue, with growth revenue and with alternating periods of falling and rising revenue.

The recent rise in inflation in Europe, caused by the pandemic, the military conflict, the increase in prices for energy resources and the violation of the logistics of energy supplies, has led to a decrease in company income. This makes it relevant to study the financial condition of companies with falling revenues. The purpose of this study is the development of tools for quantifying the impact of falling company incomes is becoming essential for making adequate management decisions. The article develops and applies an approach that allows to investigate the financial state of companies with falling revenues.

From a historical point of view, the theory of Modigliani and Miller was the first quantitative theory of the cost of capital and the structure of capital [1–3]. The authors, under a variety of assumptions, including the absence of corporate and individual taxes, the perpetuity of all companies and all cash flows, etc., obtained results that are completely different from the results of the traditional approach that existed before: the capital structure does not affect the financial performance of companies. Over the decades, many attempts have been made to modify the Modigliani–Miller theory [4–14].

Recently, Brusov et al. adapted two main theories of capital structure (Brusov–Filatova–Orekhova and Modigliani–Miller) to establish the practice of the function of companies [15–17]. This generalization takes into account the real conditions of the work of the companies. The Brusov–Filatova–Orekhova (BFO) theory, its methodology, and results are widely known (see, for example, [14, 19–26]). A lot of authors of [21–23] use the BFO theory in practice.

## THEORETICAL BACKGROUND

One of the generalization of two main theories of capital structure (Brusov–Filatova–Orekhova and

Modigliani – Miller) was coupled to account a variable revenue.

In case of Brusov–Filatova–Orekhova theory the following formula has been derived [16].

$$\frac{1 - \left( \frac{1+g}{1+WACC} \right)^n}{WACC - g} = \frac{1 - \left( \frac{1+g}{1+k_0} \right)^n}{(k_0 - g) \cdot \left[ 1 - w_d t \left[ 1 - (1+k_d)^{-n} \right] \right]}. \quad (1)$$

Here, 'n' is company age; WACC is the weighted average cost of capital; 'k<sub>0</sub>' is equity cost at L = 0; 'w<sub>d</sub>' is debt share; 'k<sub>d</sub>' is debt cost; 'g' is revenue growth rate; 't' is tax on profit rate; 'L' is the leverage level.

For perpetuity (Modigliani–Miller) limit ( $n \rightarrow \infty$ ) we arrive at the following equation for WACC in case of variable revenue [15].

$$WACC = (k_0 - g) \cdot (1 - w_d t) + g. \quad (2)$$

Formulas (1) and (2) allow study the financial state of companies with falling revenue, with growth revenue and with alternating periods of falling and rising revenue.

## RESULTS AND DISCUSSIONS

Using formulas (1) and (2), the dependences of the main financial parameters of the financial state of the company: weighted average cost of capital, WACC, discount rate, WACC – g, company value 'V', and the cost of equity 'ke' on the leverage level 'L' at different values of income falling rate 'g', different debt cost 'k<sub>d</sub>', different company ages are studied, as well as the dependence of WACC on company age 'n'. The results of calculations within Microsoft are presented below for the following financial parameters of the company: WACC, WACC – g, V, ke. Calculations are made for debt costs k<sub>d</sub> = 0.03; 0.04; 0.05 and for three company ages n = 3; 6; 9. The study was carried out for the parameters of the "Gazprom Ltd" company calculated by the authors.

### CALCULATIONS FOR DEBT COST KD = 0.03

#### Calculations of weighted average cost of capital, WACC

Below we present the results of calculations at debt cost k<sub>d</sub> = 0.03.

From Fig. 1 it is seen that all WACC(L) curves decrease with level of leverage 'L' at all 'g' values and all company ages n = 3; 6; 9. WACC values decrease with company age, but increase with increase of falling rate 'g'. Each triple of curves

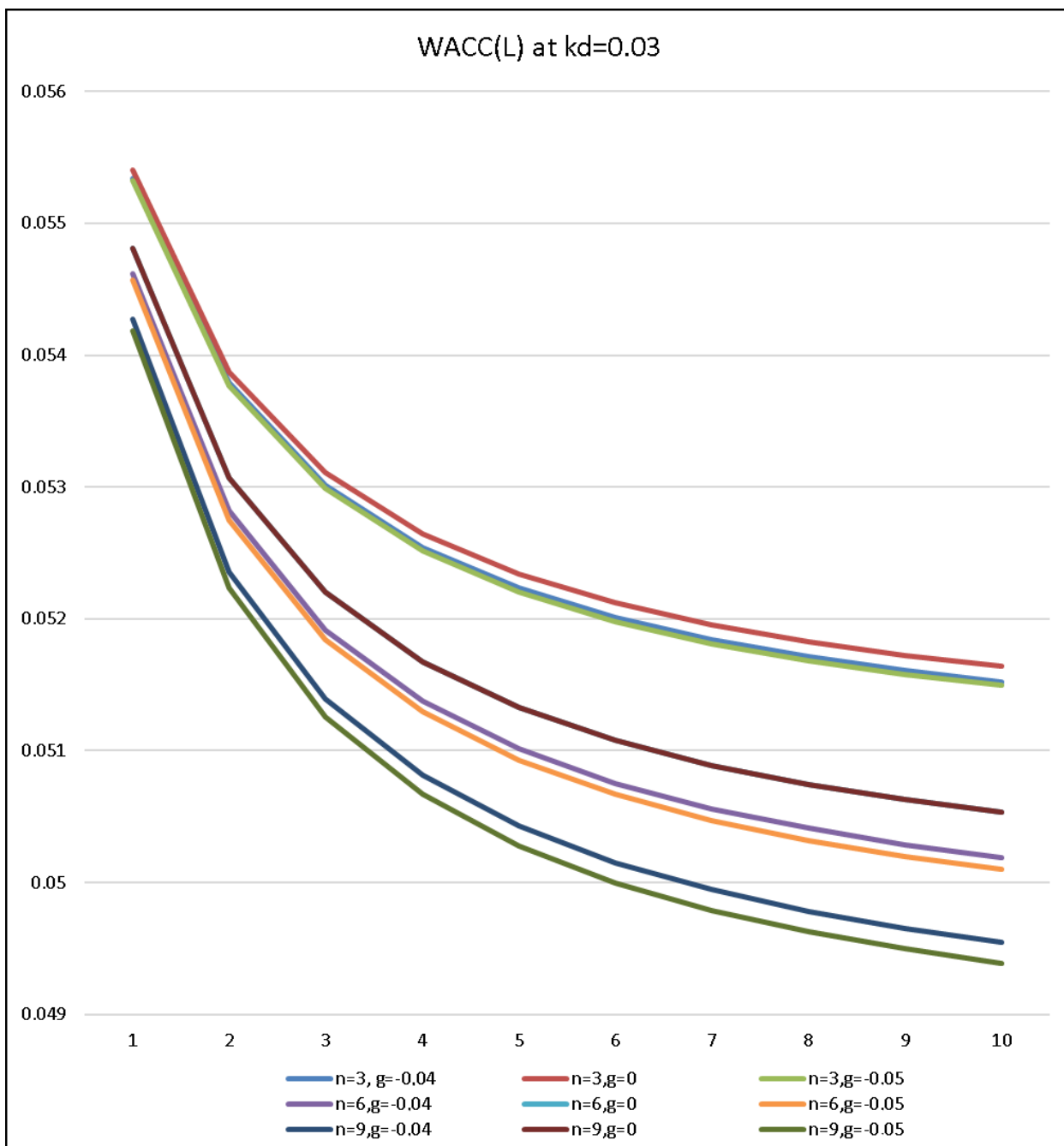


Fig. 1. The WACC Depending on the Level of Leverage L at Different Growth Rates  $g = 0; -0.04; -0.05$  and Different Company Age  $n = 3; 6; 9$

Source: Compiled by the authors.

formed for a company of a fixed age is ordered as follows (from bottom to top):  $g = -0.05; -0.04; 0$ . **The distance between the curves corresponding to different fall rates increases with the age of the company.** An increase in WACC as the rate of fall of 'g' increases indicates that WACC is no longer a discount rate, since it is intuitively clear that the discount rate must decrease as the rate of fall of g increases in order for the value of the company to rise as 'g' increases. As will be seen in

the next paragraph, the role of the discount rate is transferred to  $WACC-g$ .

#### Calculations of discount rate, $WACC-g$

From Fig. 2 it is seen that all  $(WACC-g)(L)$  curves decrease with level of leverage L at all g values and all company ages  $n = 3; 6; 9$ .  $WACC-g$  values decrease with company age 'n', as well as with increase of falling rate 'g'. Each triple of curves formed for a company of a fixed falling rate 'g' is

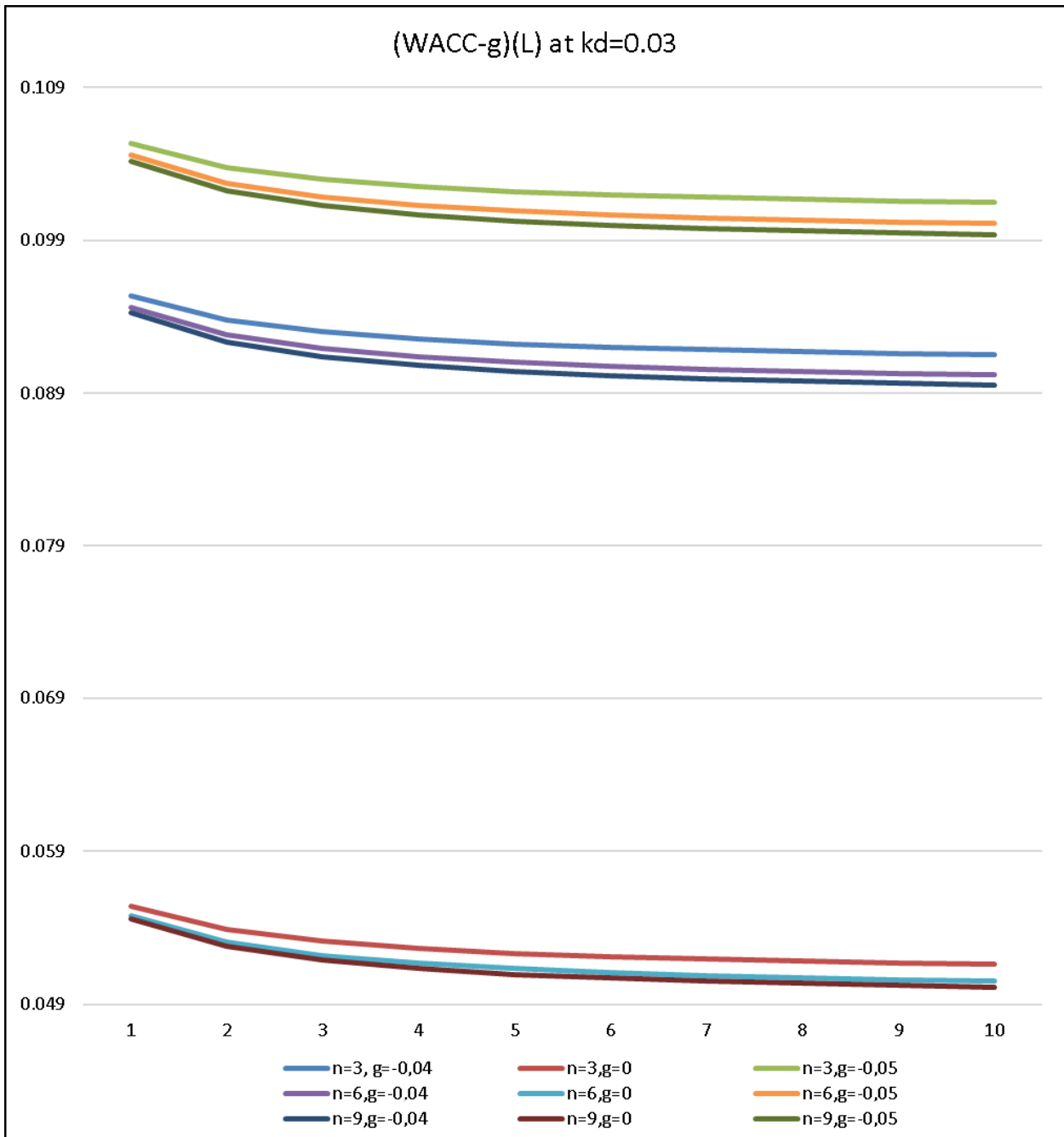


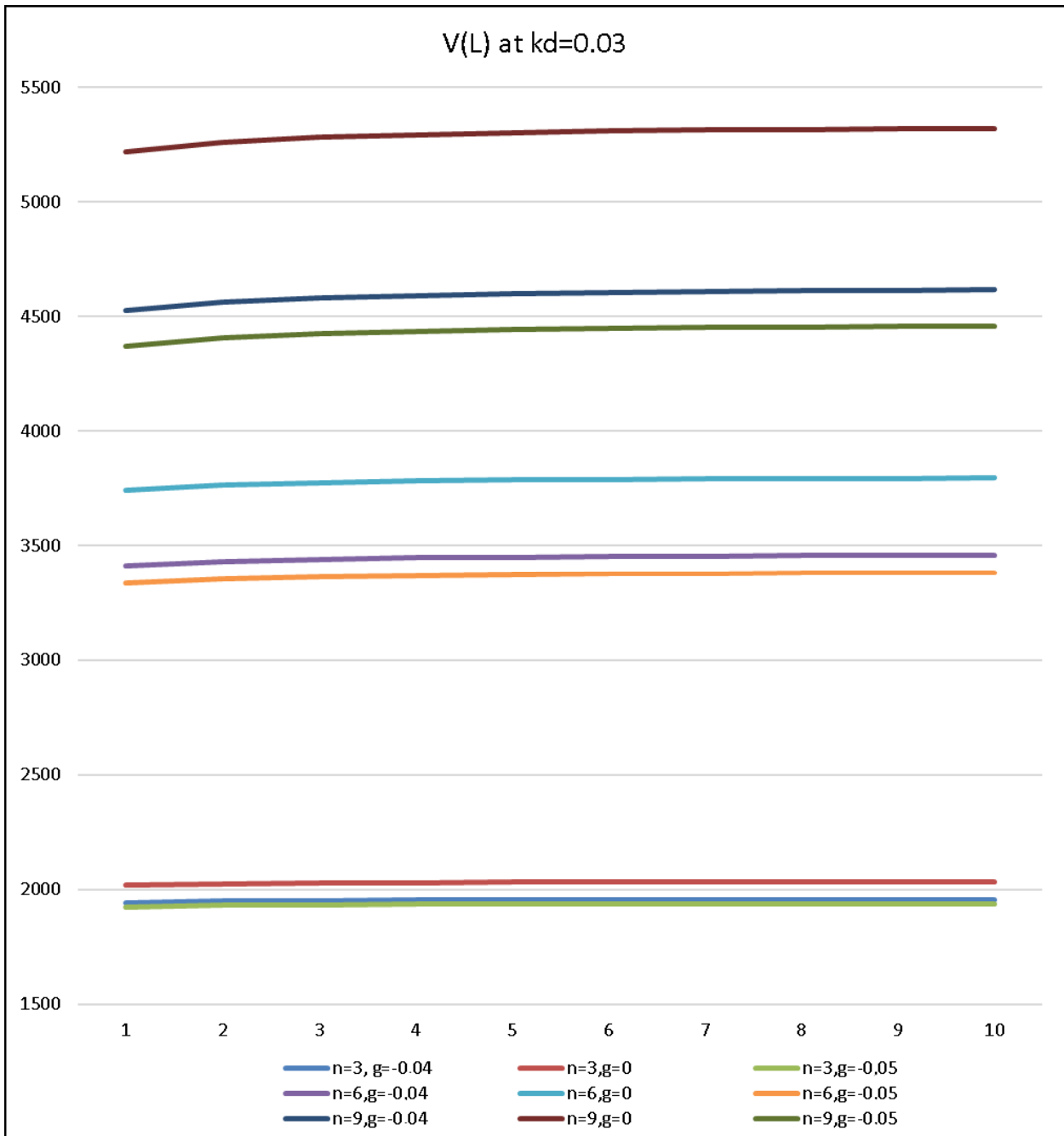
Fig. 2. The Discount Rate, WACC-g, Depending on the Level of Leverage L at Different Fall Rates  $g = 0; -0.04; -0.05$  and Different Company Age  $n = 3; 6; 9$

Source: Compiled by the authors.

ordered as follows (from bottom to top):  $n = 9; 6; 3$ . **The distance between the curves corresponding to different company ages decreases with falling rate 'g'.** A decrease in WACC-g as the rate of fall of 'g' increases indicates that WACC-g is a discount rate, because in this case the value of the company rises as 'g' increases. Thus, the role of the discount rate is transferred from WACC to WACC-g. This could be seen as well from the BFO – formula (1).

#### Calculations of company value, V

As it could be seen from Fig. 3 the company value 'V' increases with level of leverage L at all g values and all company ages  $n = 3; 6; 9$ . The company value 'V' increases with company age, as well as with increase of falling rate 'g'. Each triple of curves formed for a company of a fixed age 'n', is ordered as follows (from bottom to top):  $g = -0.05; -0.04; 0$ . **The distance between the curves corresponding to different fall rates increases with the age of the company.**



**Fig. 3. The Company Value, V, Depending on the Level of Leverage L at Different Growth Rates  $g = 0; -0.04; -0.05$  and different company age  $n = 3; 6; 9$**

Source: Compiled by the authors.

This means that influence of falling rate ‘g’ increases with company age ‘n’. For example, under decrease ‘g’ on 5% ( $g = -0.05$ ): for nine-year company ( $n = 9$ ) company value ‘V’, decreases by 12.3%, while for six-year company ( $n = 6$ ) company value ‘V’, decreases by 10.9%, and for three-year company ( $n = 3$ ) company value ‘V’, decreases by 4.7% only. An important conclusion is that the impact of the rate of decline in revenue ‘g’ on the value of company ‘V’ increases significantly with the age of company ‘n’.

**Calculations of equity cost,  $k_e$**

The equity cost ‘ $k_e$ ’, as it is seen from Fig. 4 and Fig. 5, linearly grow with level of leverage ‘L’ at all falling rate ‘g’ and all company age ‘n’. The tilt angle of the curve  $k_e(L)$  grows with ‘g’, but decrease with company age ‘n’. There is intermixture of the lines  $k_e(L)$ , corresponding to company ages six and nine years at different falling rate ‘g’. It could lead to some interesting effects,

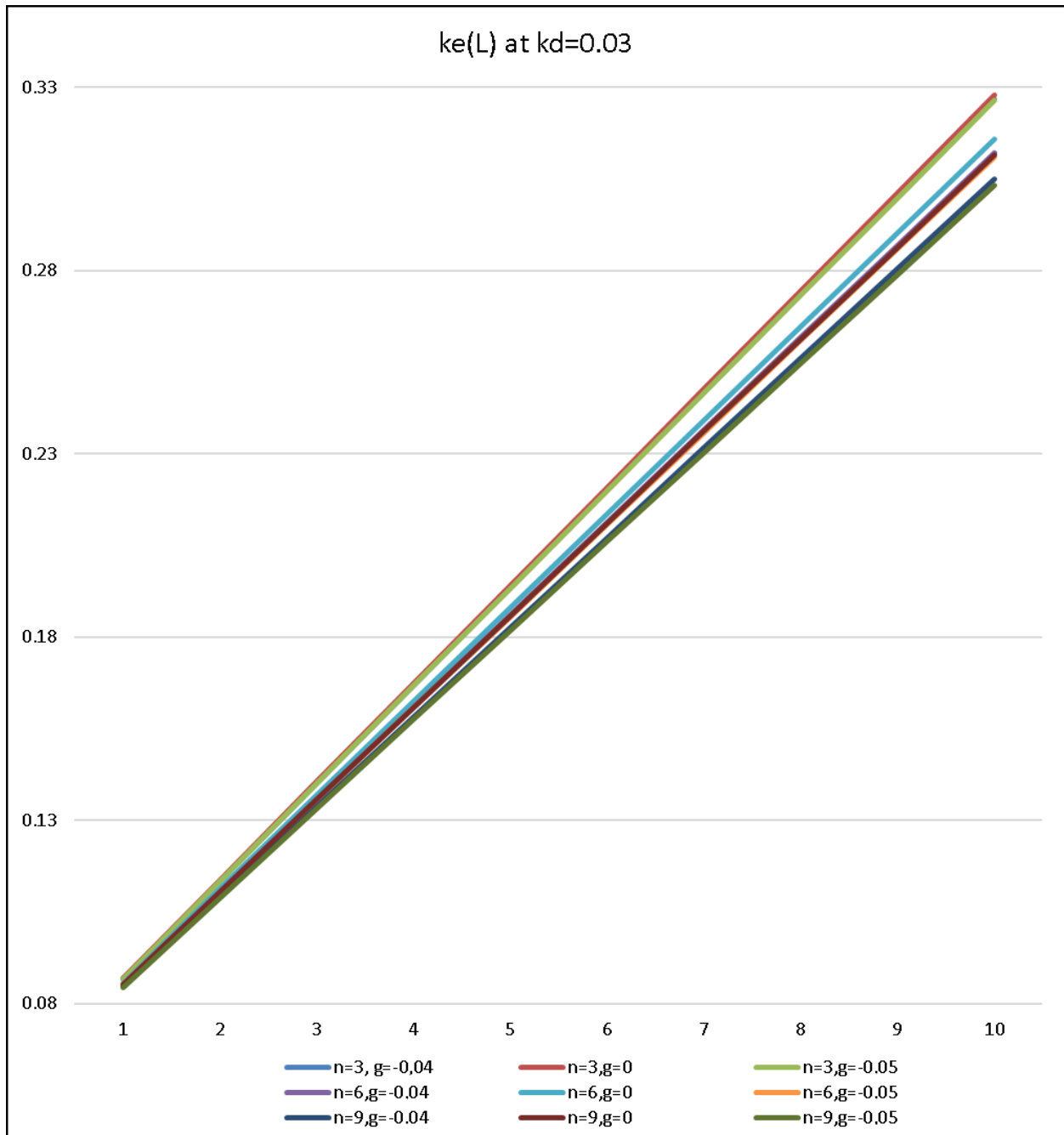


Fig. 4. The Equity Cost,  $k_e$ , Depending on the Level of Leverage  $L$  at Different Growth Rates  $g = 0; -0.04; -0.05$  and Different Company Age  $n = 3; 6; 9$

Source: Compiled by the authors.

because the cost of equity, being an economically justified amount of dividends, determines the company's dividend policy.

### CALCULATIONS FOR DEBT COST $K_D = 0.04$

#### Calculations of weighted average cost of capital, WACC

Below we present the results of calculations at debt cost  $k_d = 0.04$ .

From Fig. 6 it is seen that all WACC(L) curves decrease with level of leverage 'L' at all 'g' values and all company ages  $n = 3; 6; 9$ . WACC values decrease with company age, but increase with increase of falling rate 'g'. Each triple of curves formed for a company of a fixed age is ordered as follows (from bottom to top):  $g = -0.05; -0.04; 0$ . **The distance between the curves corresponding to different fall rates increases with the age of the company.** An increase in WACC as the rate of fall of 'g' increases

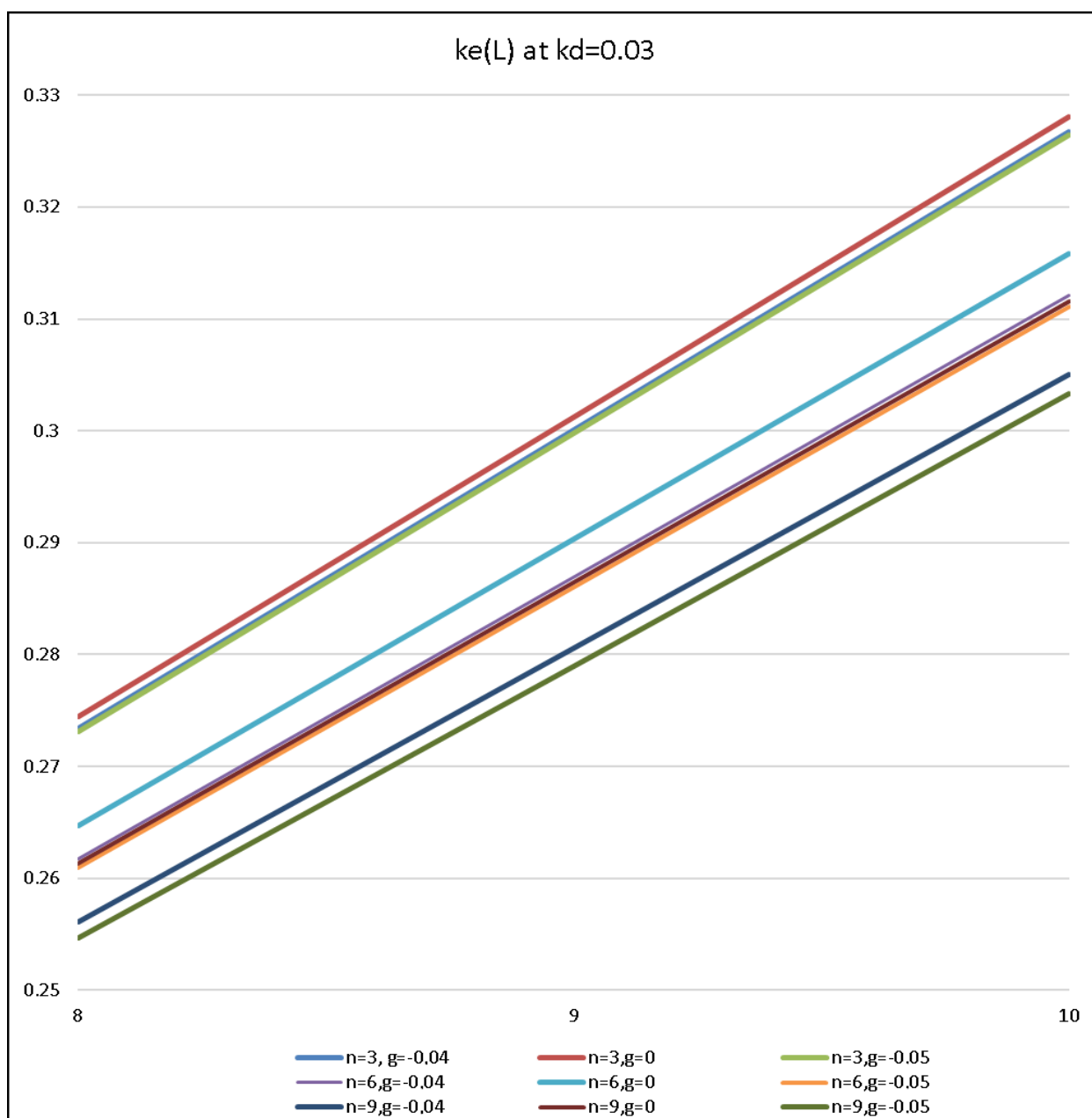


Fig. 5. The Equity Cost,  $k_e$ , Depending on the Level of Leverage  $L$  at Different Growth Rates  $g = 0; -0.04; -0.05$  and Different Company Age  $n = 3; 6; 9$  (Bigger Scale)

Source: Compiled by the authors.

indicates that WACC is no longer a discount rate, since it is intuitively clear that the discount rate must decrease as the rate of fall of 'g' increases in order for the value of the company to rise as 'g' increases. As will be seen in the next paragraph, the role of the discount rate is transferred to  $WACC-g$ .

#### Calculations of discount rate, $WACC-g$

From Fig. 7 it is seen that all ( $WACC-g$ ) ( $L$ ) curves decrease with level of leverage  $L$  at all 'g' values and all company ages  $n = 3; 6; 9$ .  $WACC-g$  values decrease with company age 'n', as well as with

increase of falling rate 'g'. Each triple of curves formed for a company of a fixed falling rate 'g' is ordered as follows (from bottom to top):  $n = 9; 6; 3$ . **The distance between the curves corresponding to different company ages decreases with falling rate 'g'.** A decrease in  $WACC-g$  as the rate of fall of  $g$  increases indicates that  $WACC-g$  is a discount rate, because in this case the value of the company rises as 'g' increases. Thus, the role of the discount rate is transferred from WACC to  $WACC-g$ . This could be seen as well from the BFO-formula (1).

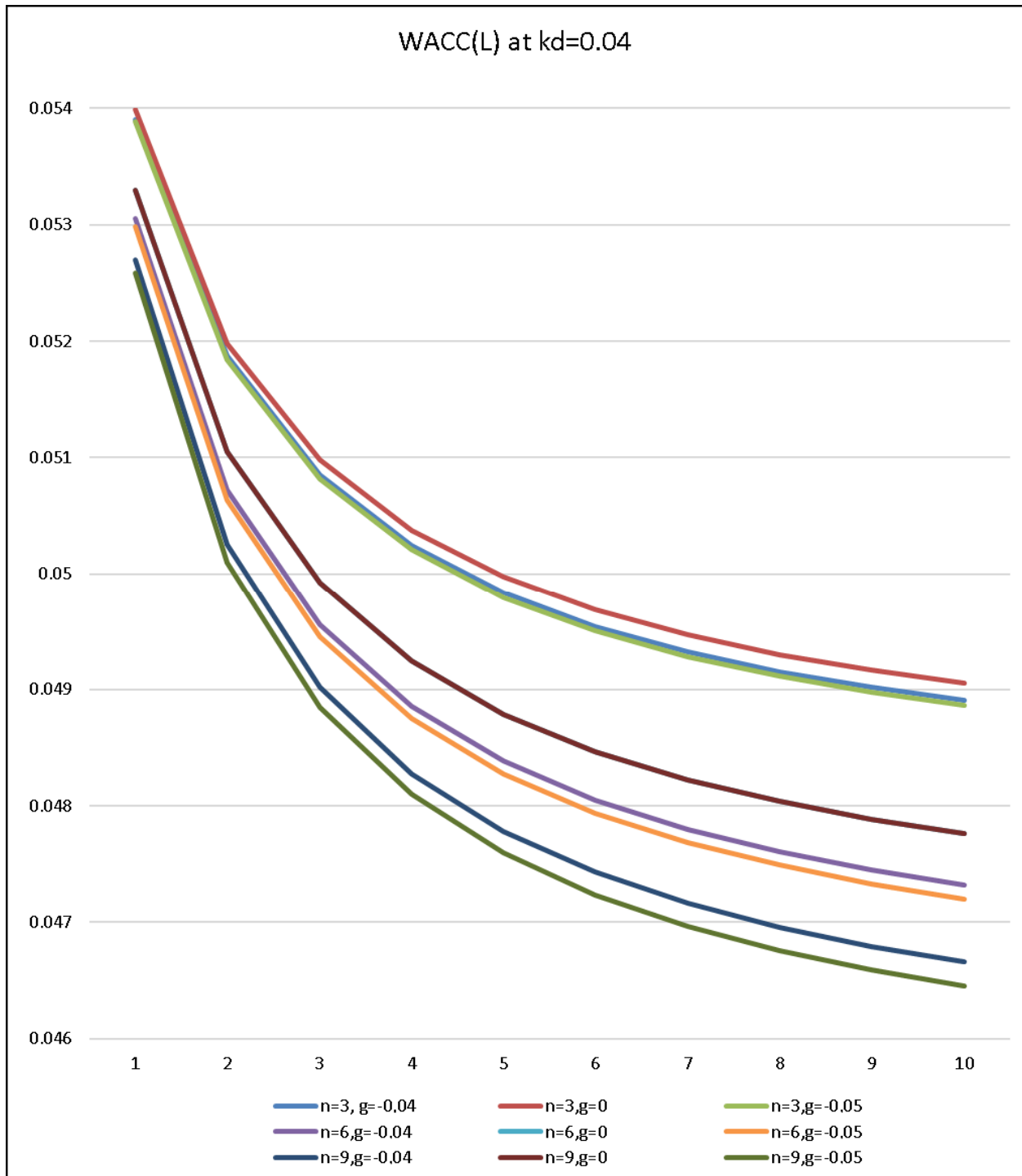


Fig. 6. The WACC Depending on the Level of Leverage L at Different Growth Rates  $g = 0; -0.04; -0.05$  and Different Company Age  $n = 3; 6; 9$

Source: Compiled by the authors.

#### Calculations of company value, V

As it could be seen from Fig. 8 the company value 'V', increase with level of leverage L at all 'g' values and all company ages  $n = 3; 6; 9$ . The company value 'V' increases with company age, as well as with increase of falling rate 'g'. Each triple of curves formed for a company of a fixed age 'n' is ordered as follows (from

bottom to top):  $g = -0.05; -0.04; 0$ . The distance between the curves corresponding to different fall rates increases with the age of the company. This means that influence of falling rate 'g' increases with company age 'n'. For example, under decrease 'g' on 5% ( $g = -0.05$ ): for nine-year company ( $n = 9$ ) company value 'V' decreases by 12.3%, while for six-



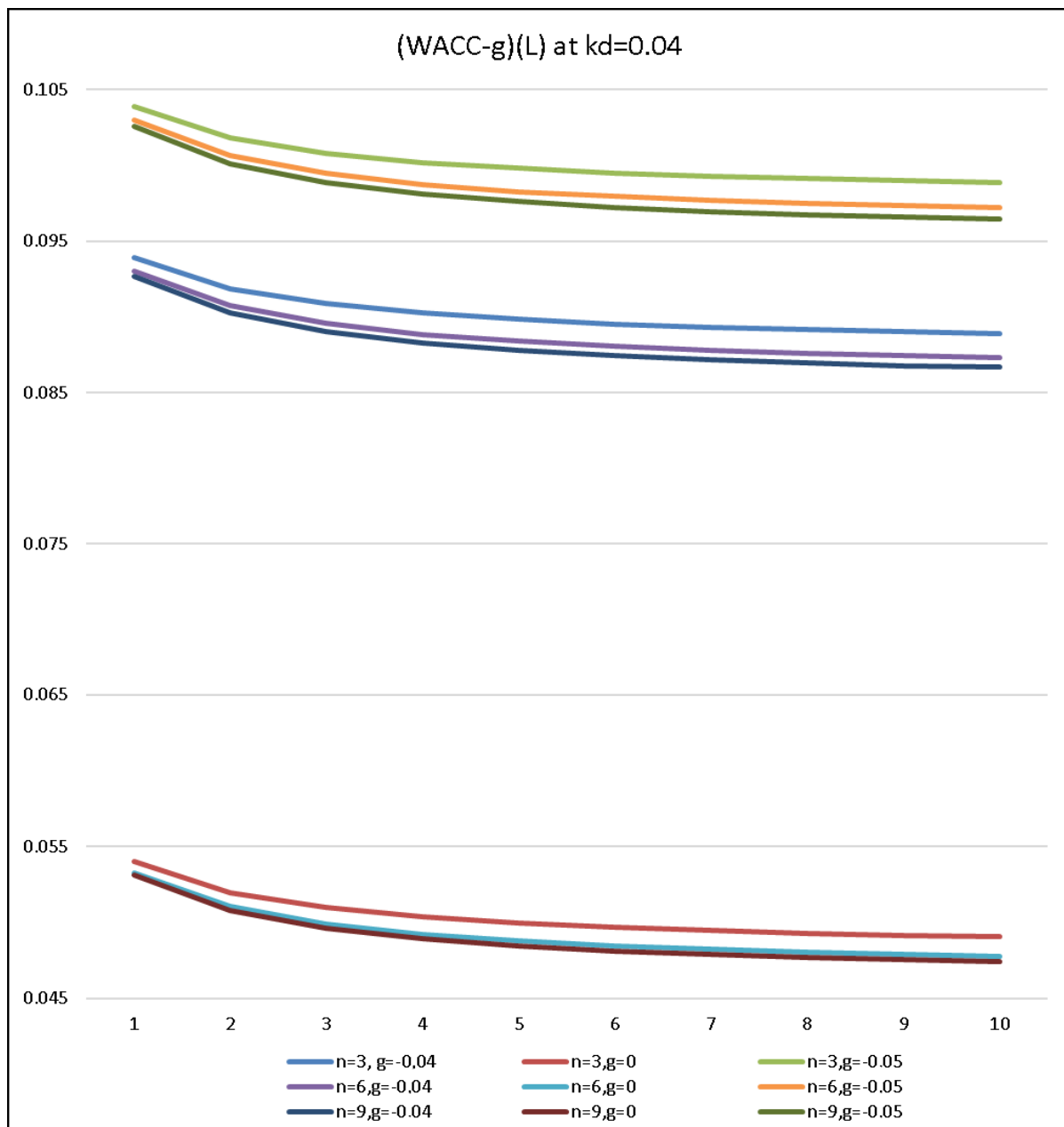


Fig. 7. The Discount Rate, WACC-g, Depending on the Level of Leverage L at Different Growth Rates g = 0; -0.04; -0.05 and Different Company Age n = 3; 6; 9

Source: Compiled by the authors.

year company (n = 6) company value 'V' decreases by 10.9%, and for three-year company (n = 3) company value 'V' decreases by 4.7% only. An important conclusion is that the impact of the rate of decline in revenue g on the value of company 'V' increases significantly with the age of company 'n'.

**Calculations of equity cost, ke**

The equity cost, ke, as it is seen from Fig. 9 and Fig. 10, linearly grow with level of leverage L at all falling rate 'g' and all company age 'n'. The tilt angle of the curve ke (L) grows with 'g', but decrease with company age 'n'.

There is intermixture of the lines ke (L), corresponding to company ages six and nine years at different falling rate 'g'. It could lead to some interesting effects, because the cost of equity, being an economically justified amount of dividends, determines the company's dividend policy.

**CALCULATIONS FOR DEBT COST KD = 0.05**

**Calculations of weighted average cost of capital, WACC**

Below we present the results of calculations at debt cost kd = 0.05.

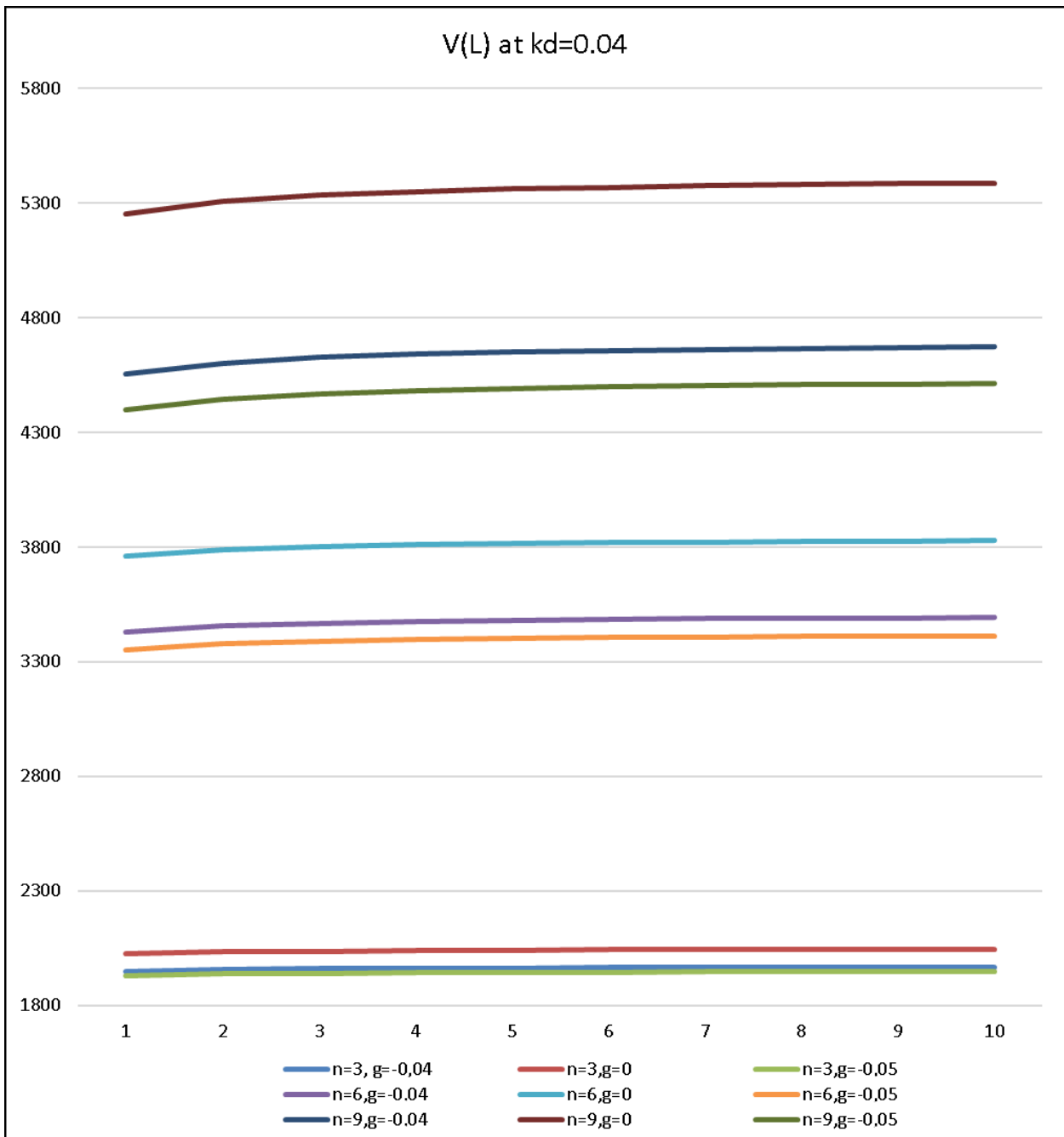


Fig. 8. The Company Value, V, Depending on the Level of Leverage L at Different Growth Rates  $g = 0; -0.04; -0.05$  and Different Company Age  $n = 3; 6; 9$

Source: Compiled by the authors.

From Fig. 11 it is seen that all WACC(L) curves decrease with level of leverage L at all 'g' values and all company ages  $n = 3; 6; 9$ . WACC values decrease with company age, but increase with increase of falling rate 'g'. Each triple of curves formed for a company of a fixed age is ordered as follows (from bottom to top):  $g = -0.05; -0.04; 0$ . **The distance between the curves corresponding to different fall rates increases with the age of the company.** An increase in WACC as the rate of fall of 'g' increases

indicates that WACC is no longer a discount rate, since it is intuitively clear that the discount rate must decrease as the rate of fall of 'g' increases in order for the value of the company to rise as 'g' increases. As will be seen in the next paragraph, the role of the discount rate is transferred to WACC-g.

#### Calculations of discount rate, WACC-g

From Fig. 12 it is seen that all (WACC-g) (L) curves decrease with level of leverage L at all 'g' values

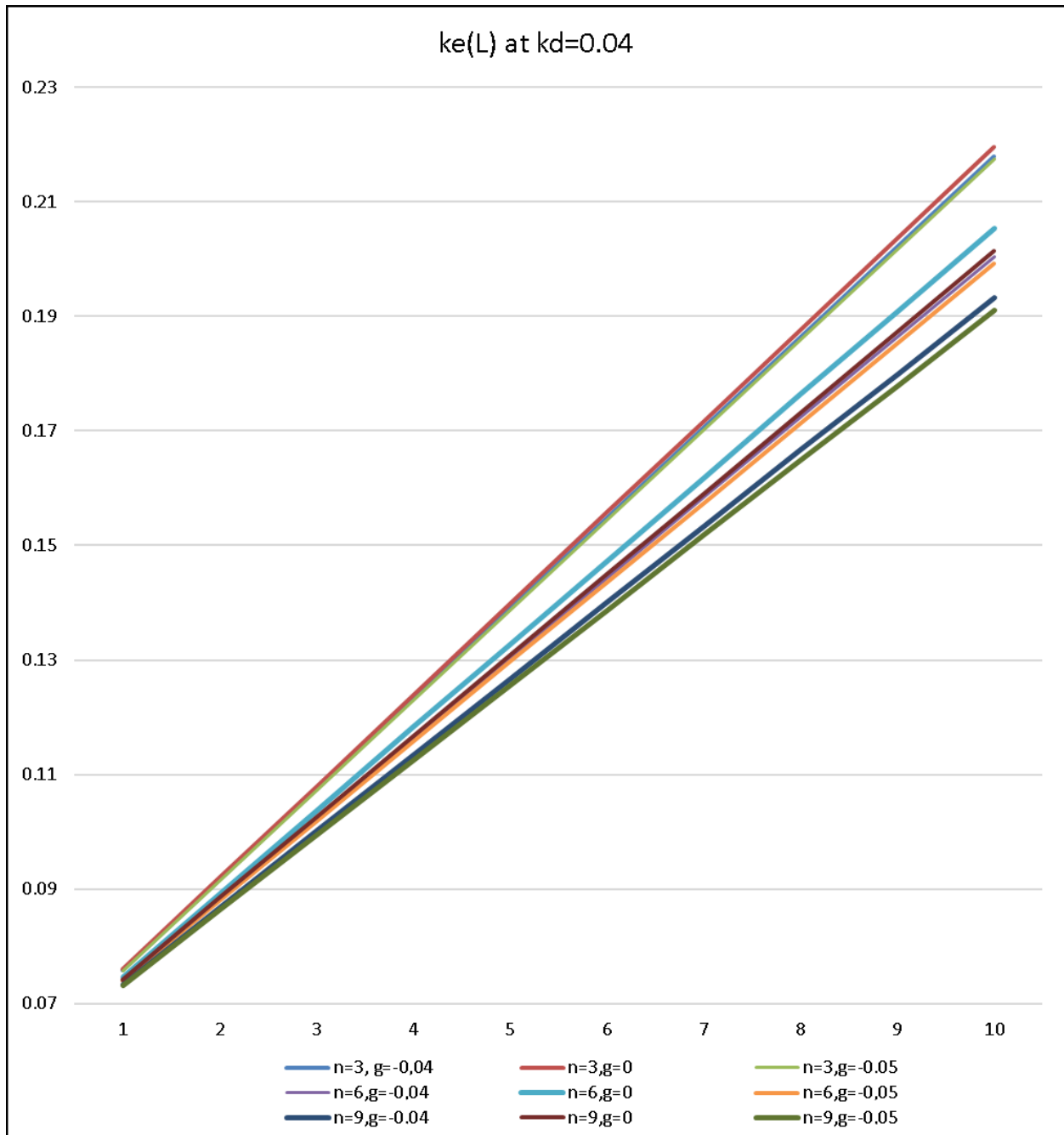


Fig. 9. The Equity Cost, Ke, Depending on the Level of Leverage L at Different Growth Rates  $g = 0; -0.04; -0.05$  and Different Company Age  $n = 3; 6; 9$

Source: Compiled by the authors.

and all company ages  $n = 3; 6; 9$ . WACC-g values decrease with company age 'n', as well as with increase of falling rate 'g'. Each triple of curves formed for a company of a fixed falling rate g is ordered as follows (from bottom to top):  $n = 9; 6; 3$ . **The distance between the curves corresponding to different company ages decreases with falling rate 'g'.** A decrease in WACC-g as the rate of fall of 'g' increases indicates that WACC-g is a discount rate, because in this case the value of the company

rises as 'g' increases. Thus, the role of the discount rate is transferred from WACC to WACC-g. This could be seen as well from the BFO-formula (1).

#### Calculations of company value, V

As it could be seen from Fig. 13 the company value, V, increase with level of leverage L at all 'g' values and all company ages  $n = 3; 6; 9$ . The company value 'V' increases with company age, as well as with increase of falling rate 'g'. Each triple of curves

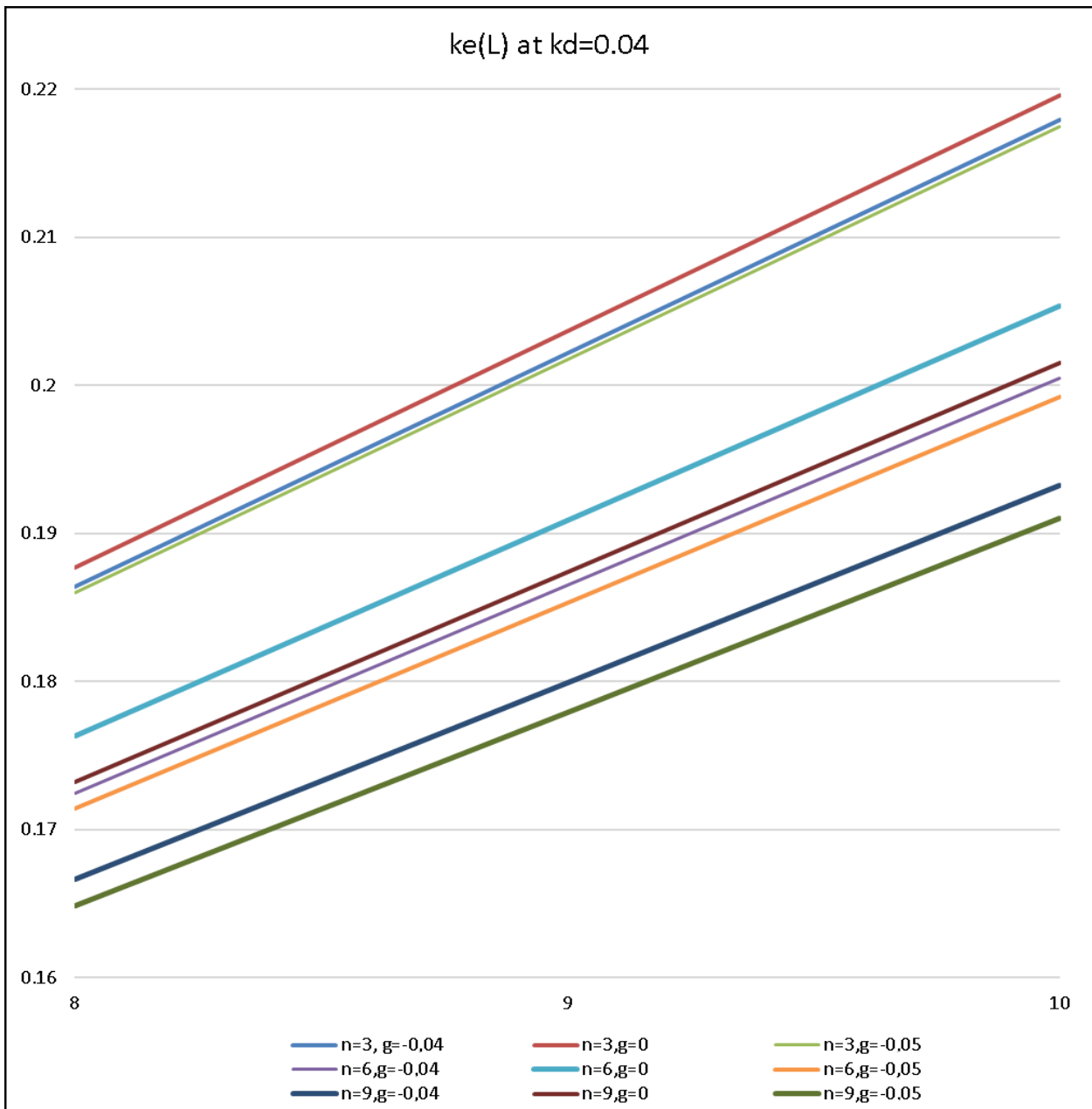


Fig. 10. The Equity Cost,  $K_e$ , Depending on the Level of Leverage  $L$  at Different Growth Rates  $g = 0; -0.04; -0.05$  and Different Company Age  $n = 3; 6; 9$  (Bigger Scale)

Source: Compiled by the authors.

formed for a company of a fixed age 'n', is ordered as follows (from bottom to top):  $g = -0.05; -0.04; 0$ . **The distance between the curves corresponding to different fall rates increases with the age of the company.** This means that influence of falling rate 'g' increases with company age 'n'. For example, under decrease  $g$  on 5% ( $g = -0.05$ ): for nine-year company ( $n = 9$ ) company value 'V' decreases by 16.2%, while for six-year company ( $n = 6$ ) company value 'V' decreases by 10.8%, and for three-year company ( $n = 3$ ) company value 'V' decreases by 4.7% only. An important conclusion is that the

impact of the rate of decline in revenue 'g' on the value of company 'V' increases significantly with the age of company 'n'.

Below we compare the results for company value  $V$  at  $k_d = 0.03; 0.04$  and  $0.05$  under decrease 'g' on 5% ( $g = -0.05$ ).

At  $k_d = 0.03; 0.04$  and  $0.05$  under decrease 'g' on 5% ( $g = -0.05$ ) decrease of **nine-year** company value is equal to 16.2%; 16.2%; 12.3%;

At  $k_d = 0.03; 0.04$  and  $0.05$  under decrease 'g' on 5% ( $g = -0.05$ ) decrease of **six-year** company value is equal to 10.9%; 10.9%; 10.8%;

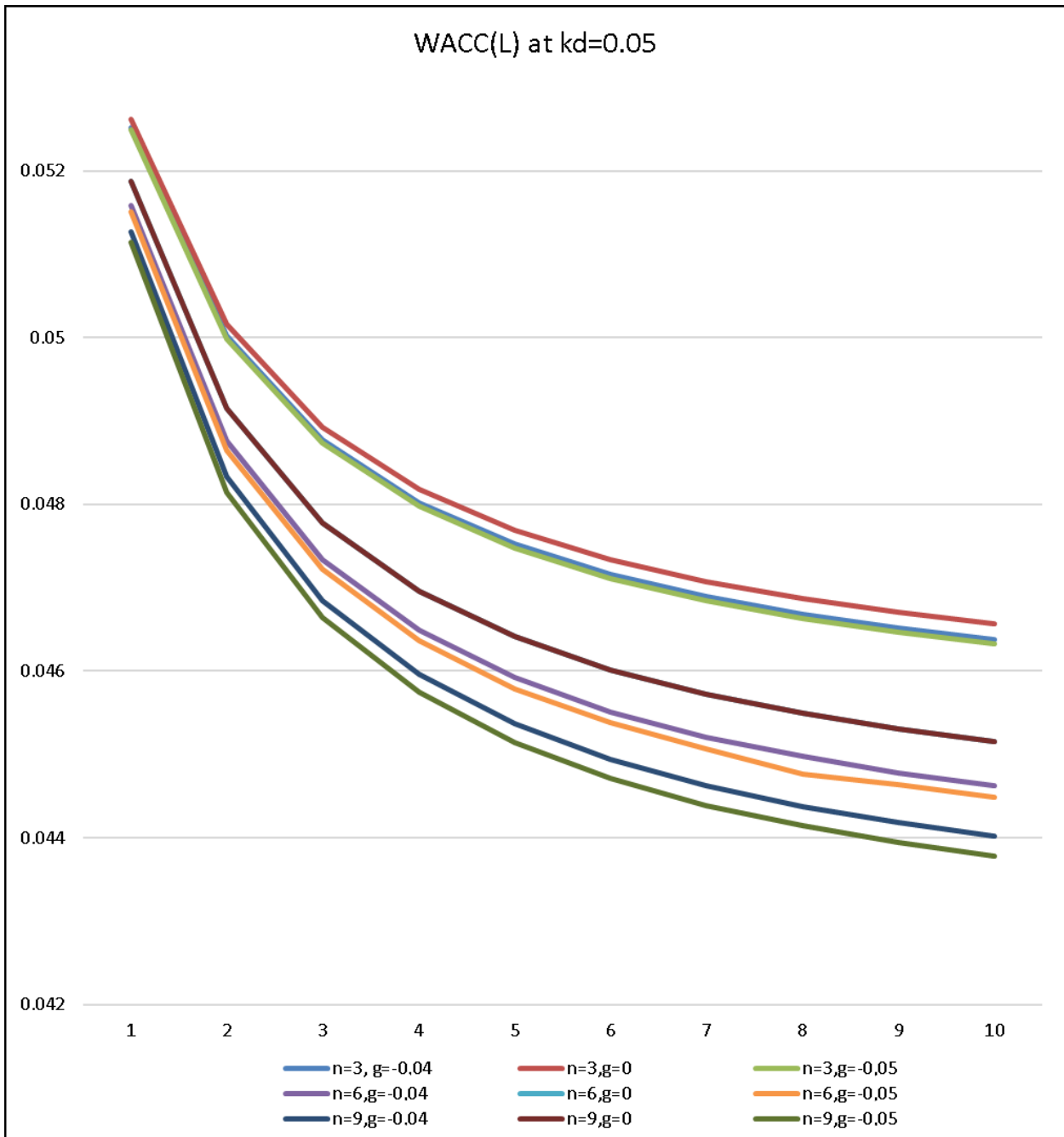


Fig. 11. The WACC Depending on the Level of Leverage L at Different Growth Rates  $g = 0; -0.04; -0.05$  and Different Company Age  $n = 3; 6; 9$

Source: Compiled by the authors.

At  $k_d = 0.03; 0.04$  and  $0.05$  under decrease 'g' on 5% ( $g = -0.05$ ) decrease of three-year company value is equal to 4.7%; 3.8%; 4.7%.

**Calculations of equity cost,  $k_e$**

The equity cost,  $k_e$ , as it is seen from Fig. 14 and Fig. 15, linearly grows with level of leverage L at all falling rate g and all company age 'n'. The tilt angle of the curve  $k_e(L)$  grows with 'g', but decrease with company age 'n'. There is intermixture of the lines

$k_e(L)$ , corresponding to company ages six and nine years at different falling rate 'g'. It could lead to some interesting effects, because the cost of equity, being an economically justified amount of dividends, determines the company's dividend policy.

**Dependence of Weighted Average Cost of Capital, WACC, on Company Age, n**

Below we study the dependence of weighted average cost of capital, WACC, on company age 'n'

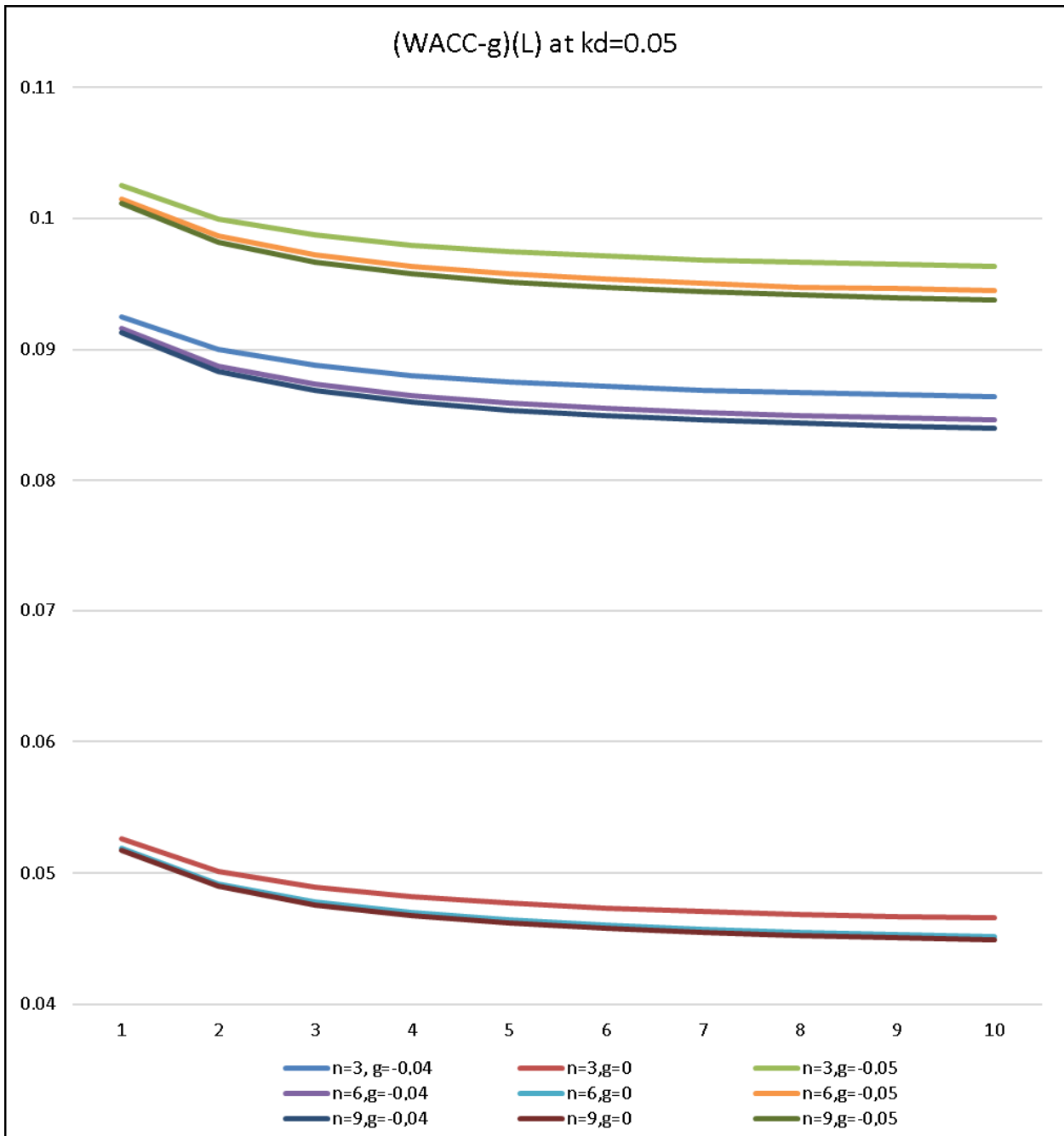


Fig. 12. The discount rate, WACC-g, B=Depending on the Level of Leverage L at Different Growth Rates  $g = 0; -0.04; -0.05$  and Different Company Age  $n = 3; 6; 9$

Source: Compiled by the authors.

at two values of falling rate,  $g = -0.04$  and  $g = -0.05$ , at three values of  $k_d$  (0.03; 0.04; 0.05) and two values of leverage level  $L = 0.5$  and 0.1. We try to clarify whether the golden age effect takes place at these particular values of equity and debt costs ( $k_0 = 6\%$ ;  $k_d = 3\%$ ).

It was shown for the first time [17] that the value of WACC in the Modigliani – Miller theory is not minimal, and the value of the company is not maximal, contrary to the opinion of all financiers: it turned out that at

a certain age of the company, the value of WACC is lower than in the eternal Modigliani – Miller theory, and the value of company ‘V’ is greater than the value of company ‘V’ in the Modigliani – Miller theory.

From Fig. 16 it is seen that WACC decreases with  $n$  monotonically at all three values of  $k_d$  (0.03; 0.04; 0.05) and tends to perpetuity (Modigliani – Mille) limit, which could be calculated by the use of Modigliani – Miller formula for WACC, generalized by us to the case of variable income:

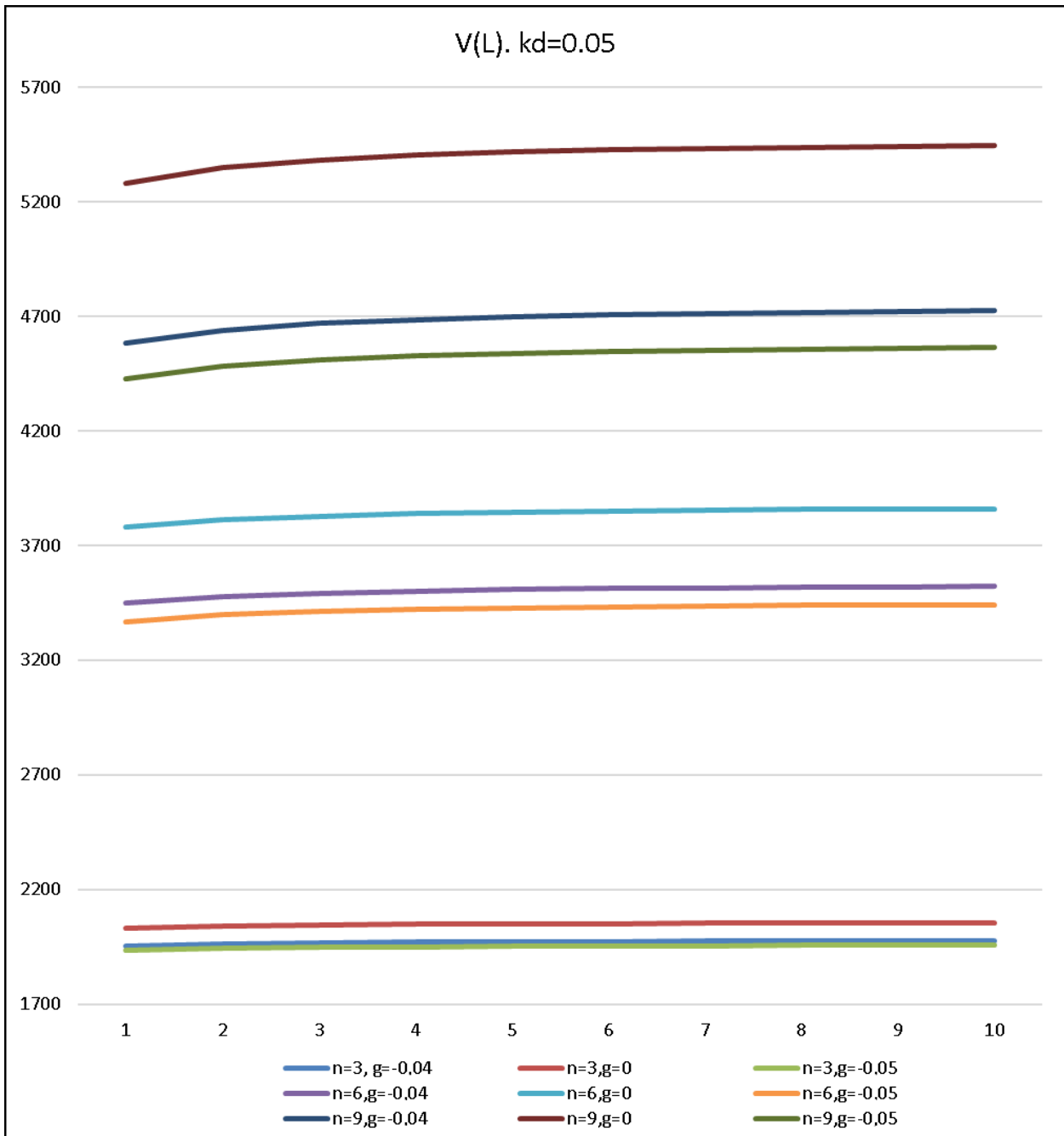


Fig. 13. The Company Value, V, Depending on the Level of Leverage L at Different Growth Rates  $g = 0; -0.04; -0.05$  and Different Company Age  $n = 3; 6; 9$

Source: Compiled by the authors.

$$WACC = (k_0 - g) \cdot (1 - w_d t) + g \quad (3)$$

WACC = 5.33% at L = 0.5.

Note, that WACC decreases in debt value, kd (0.03; 0.04; 0.05). All three curves WACC(n) tends to the same limit WACC = 5.33%, because, as it is seen from the formula (3) WACC in perpetuity (Modigliani – Mille) limit does not depend on debt cost, kd.

From Fig. 17 it is seen that WACC decreases with n monotonically at all three values of kd (0.03;

0.04; 0.05) and tends to perpetuity (Modigliani – Mille) limit, which could be calculated by the use of Modigliani – Miller formula (3) for WACC, generalized by us to the case of variable income:

$$WACC = 5\% \text{ at } L = 1.$$

Note, that WACC decreases in debt value, kd (0.03; 0.04; 0.05). All three curves WACC(n) tends to the same limit WACC = 5%, because, as it is seen from the formula (3) WACC in perpetuity (Modigliani – Mille) limit does not depend on debt cost, kd.

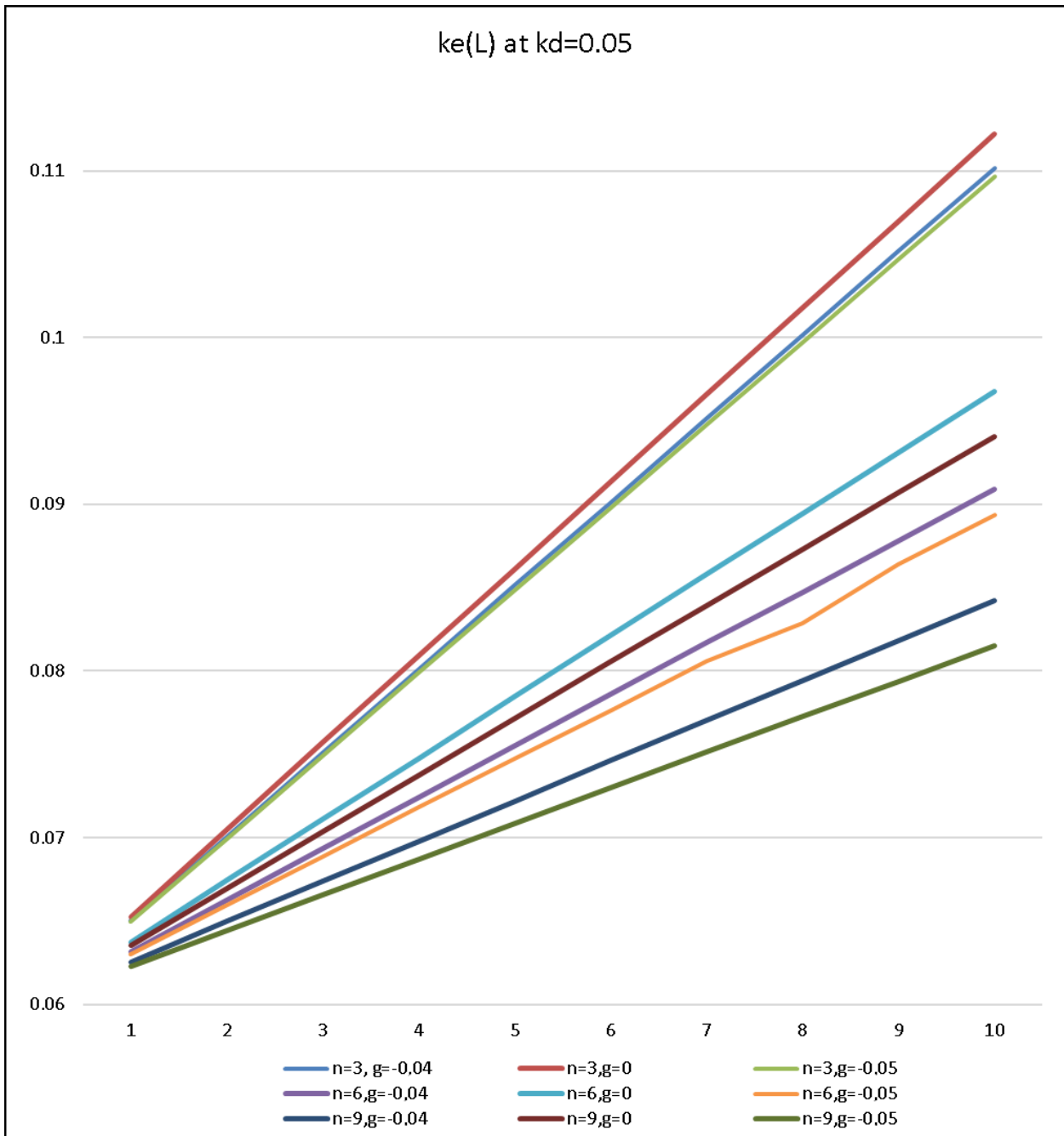


Fig. 14. The Equity Cost,  $k_e$ , Depending on the Level of Leverage  $L$  at Different Growth Rates  $g = 0; -0.04; -0.05$  and Different Company Age  $n = 3; 6; 9$

Source: Compiled by the authors.

From Fig. 19 it is seen that WACC decreases with  $n$  monotonically at all three values of  $k_d$  (0.03; 0.04; 0.05) and tends to perpetuity (Modigliani – Mille) limit, which could be calculated by the use of Modigliani – Miller formula (3) for WACC, generalized by us to the case of variable income:

$$WACC = 4.9\% \text{ at } L = 0.5.$$

Note, that WACC decreases with debt value,  $k_d$  (0.03; 0.04; 0.05). All three curves WACC( $n$ ) tends

to the same limit WACC = 4.9%, because, as it is seen from the formula (3) WACC in perpetuity (Modigliani – Mille) limit does not depend on debt cost,  $k_d$ .

We conclude, that the “Golden age” effect is absent at these particular values of equity and debt costs ( $k_0 = 6\%$ ;  $k_d = 3\%$ ), because WACC decreases with ‘ $n$ ’ monotonically at all three values of  $k_d$  (0.03; 0.04; 0.05) at all leverage level values (0.5 and 1) and both falling rates ‘ $g$ ’ (-0.04 and -0.05).



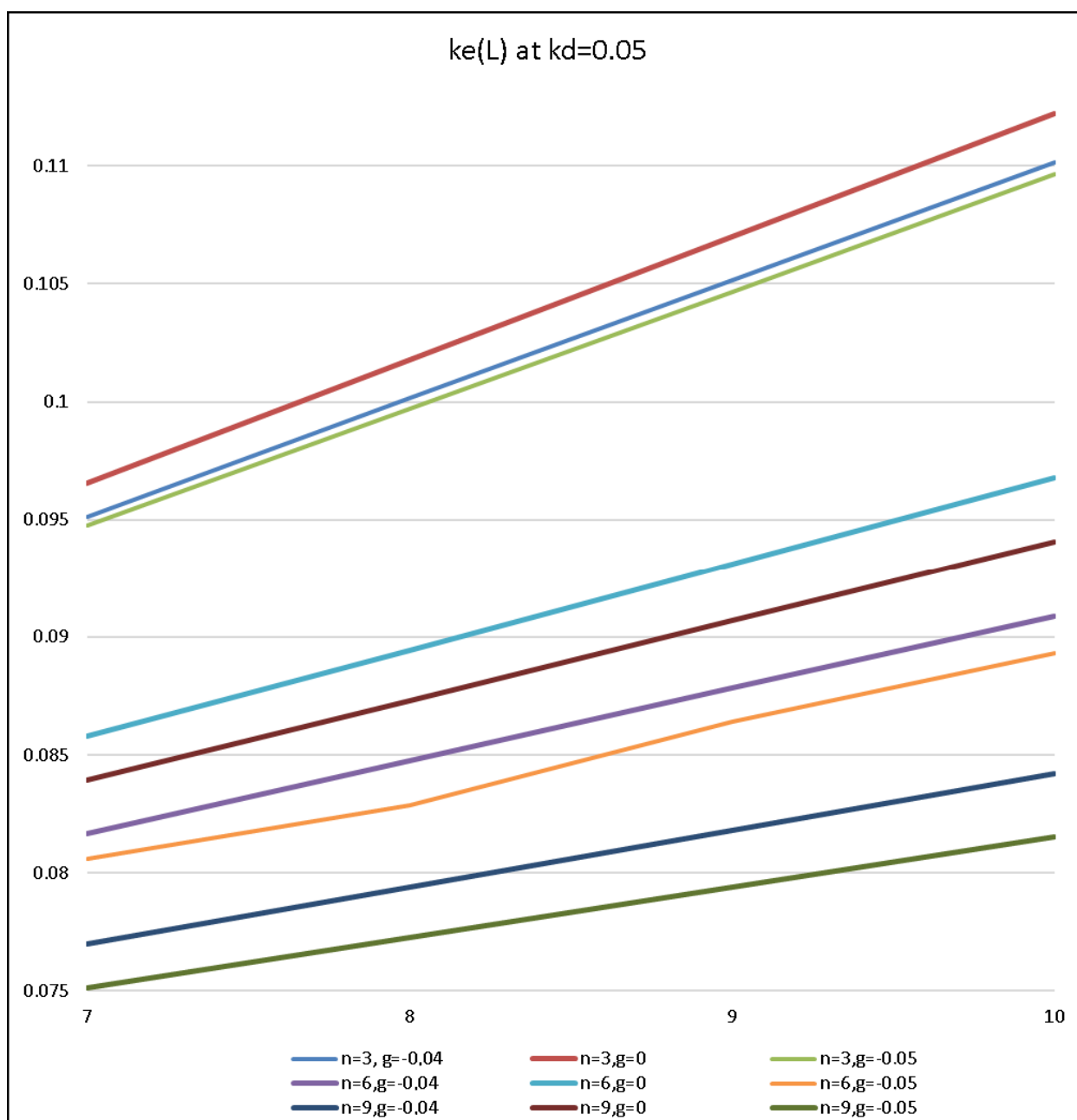


Fig. 15. The Equity Cost,  $k_e$ , Depending on the Level of Leverage  $L$  at Different Growth Rates  $g = 0; -0.04; -0.05$  and Different Company Age  $n = 3; 6; 9$  (Bigger Scale)

Source: Compiled by the authors.

## CONCLUSIONS

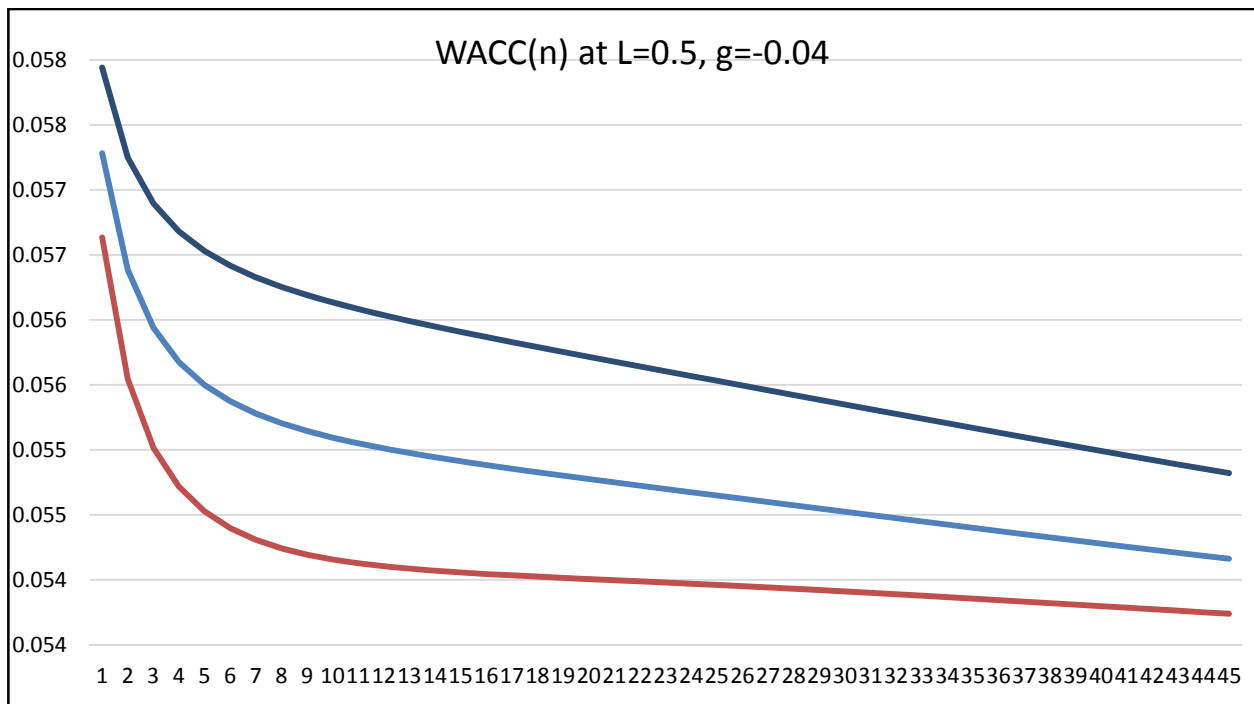
The article develops and applies an approach that allows to investigate the financial state of companies with falling revenues.

From Fig. 1, 5, 9 it is seen that all  $WACC(L)$  curves decrease with level of leverage  $L$  at all 'g' values and all company ages  $n = 3; 6; 9$ .  $WACC$  values decrease with company age, but increase with increase of falling rate 'g'. Each triple of curves formed for a company of a fixed age is ordered as follows (from bottom to top):  $g = -0.05; -0.04; 0$ . **The distance between the curves**

**corresponding to different fall rates increases with the age of the company.** An increase in  $WACC$  as the rate of fall of 'g' increases indicates that  $WACC$  is no longer a discount rate, since it is intuitively clear that the discount rate must decrease as the rate of fall of 'g' increases in order for the value of the company to rise as 'g' increases. As will be seen in the next paragraph, the role of the discount rate is transferred to  $WACC-g$ .

From Fig. 2, 6, 10 it is seen that all  $(WACC-g)(L)$  curves decrease with level of leverage 'L' at all g values and all company ages  $n = 3; 6; 9$ .  $WACC-g$  values

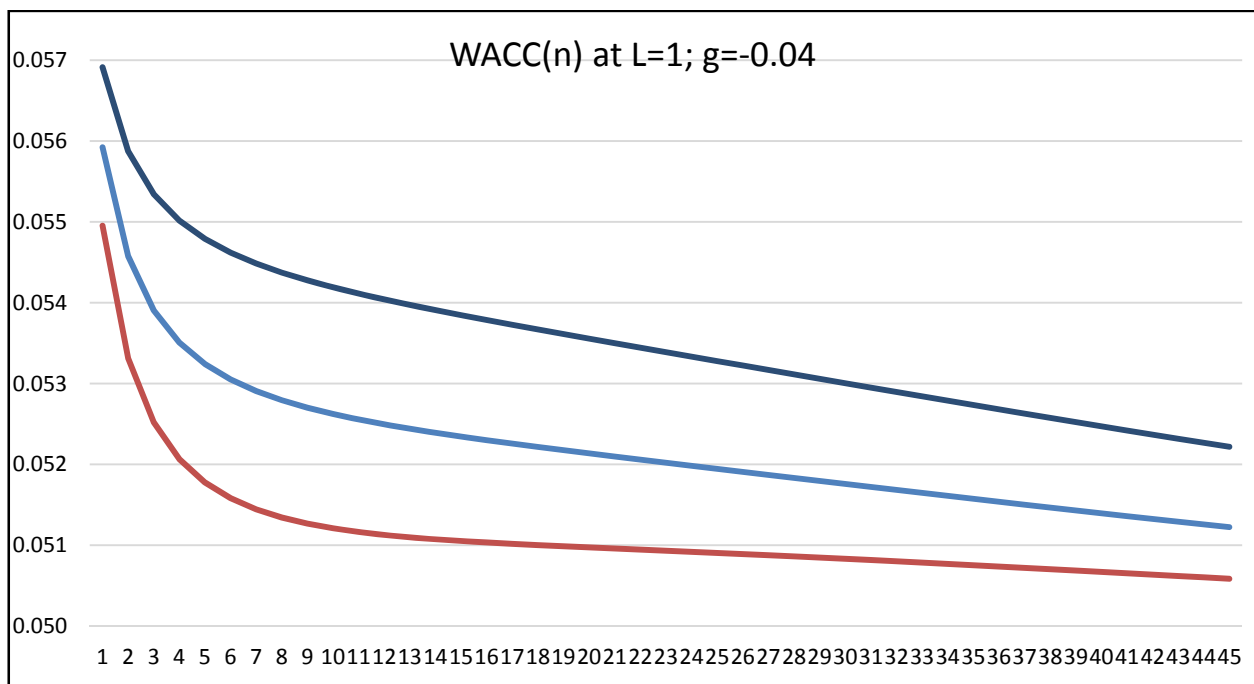
**Dependence of weighted average cost of capital, WACC, on company age at falling rate  $g = -0.04$  and leverage level  $L = 0.5$**



**Fig. 16. The WACC Depending on Company Age  $n$  at Different Debt Cost  $k_d = 0.05; 0.04; 0.03$  (from Bottom to Top); at Leverage Level  $L = 0.5$  and  $g = -0.04$**

Source: Compiled by the authors.

**Dependence of weighted average cost of capital, WACC, on company age at falling rate  $g = -0.04$  and leverage level  $L = 0.1$**



**Fig. 17. The WACC Depending on Company Age  $n$  at Different Debt Cost  $k_d = 0.05; 0.04; 0.03$  (From Bottom to Top); at Leverage Level  $L = 1$  and  $g = -0.04$**

Source: Compiled by the authors.

Dependence of weighted average cost of capital, WACC, on company age at falling rate  $g = -0.05$  and leverage level  $L = 0.5$

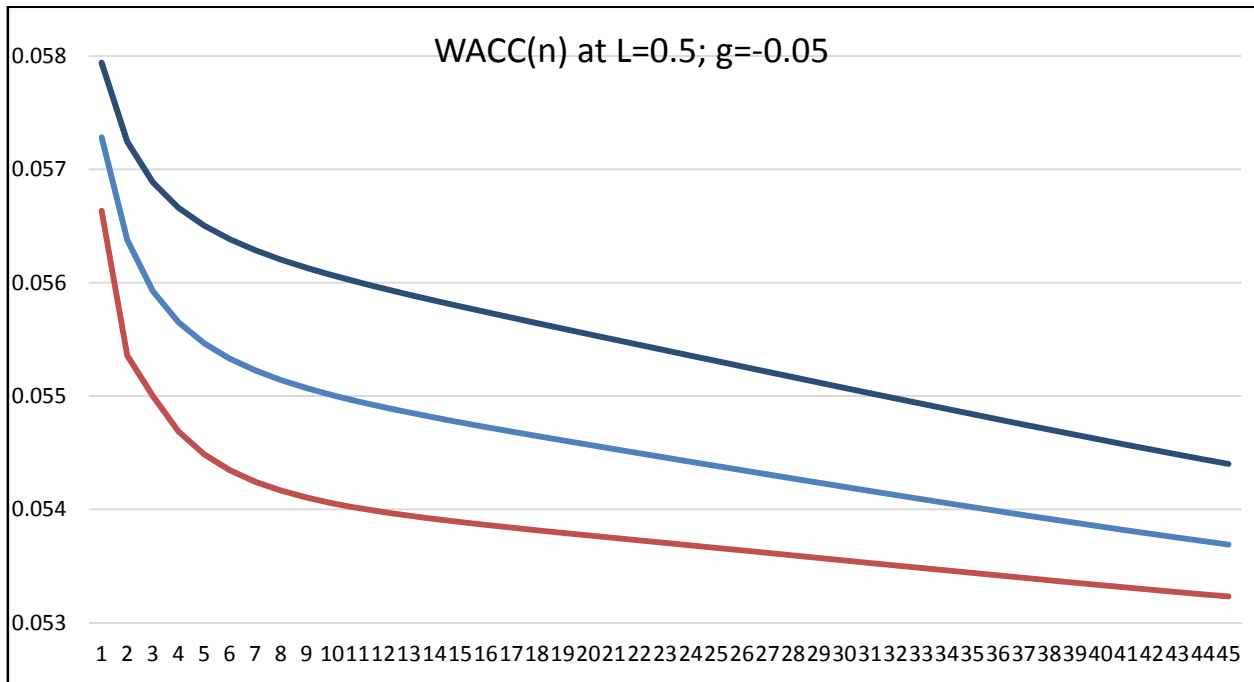


Fig. 18. The WACC Depending on Company Age  $n$  at Different Debt Cost  $k_d = 0.05; 0.04; 0.03$  (from Bottom to Top); at Leverage Level  $L = 0.5$  and  $g = -0.05$

Source: Compiled by the authors.

Dependence of weighted average cost of capital, WACC, on company age at falling rate  $g = -0.05$  and leverage level  $L = 1$

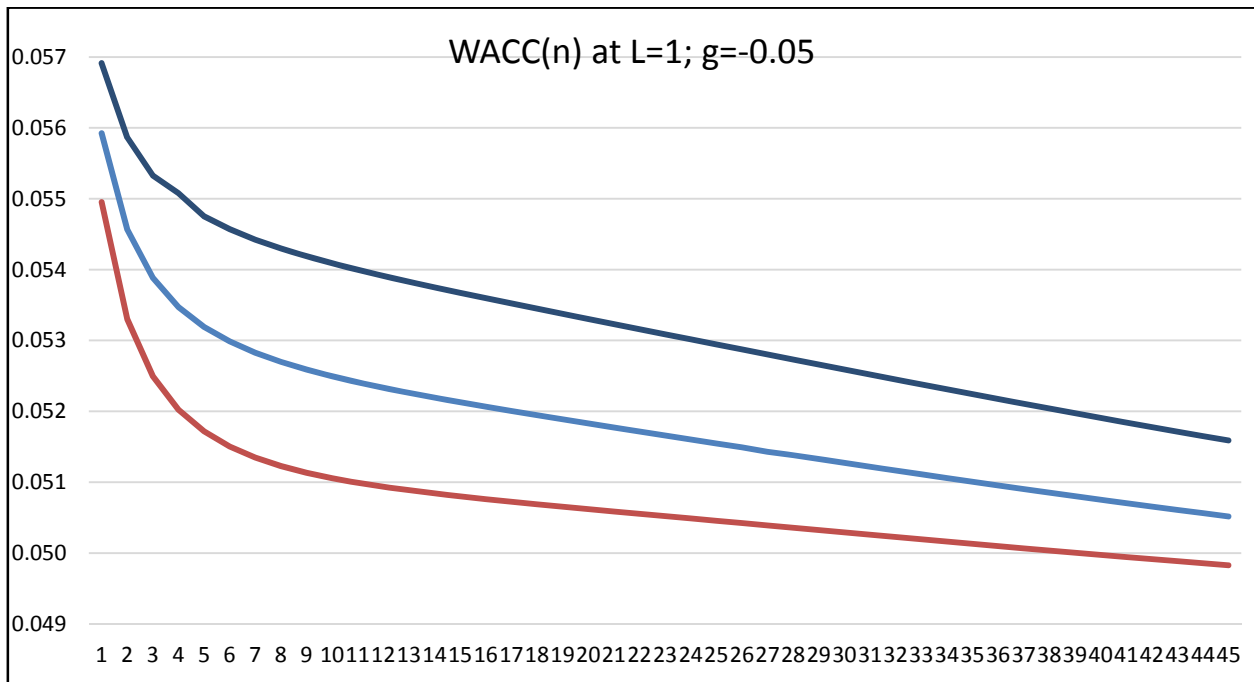


Fig. 19. The WACC Depending on Company Age  $n$  at Different Debt Cost  $k_d = 0.05; 0.04; 0.03$  (from Bottom to Top); at Leverage Level  $L = 1$  and  $g = -0.05$

Source: Compiled by the authors.

decreases with company age 'n', as well as with increase of falling rate 'g'. Each triple of curves formed for a company of a fixed falling rate 'g' is ordered as follows (from bottom to top): n = 9; 6; 3. **The distance between the curves corresponding to different company ages decreases with falling rate 'g'**. A decrease in WACC-g as the rate of fall of g increases indicates that WACC-g is a discount rate, because in this case the value of the company rises as 'g' increases. Thus, the role of the discount rate is transferred from WACC to WACC-g. This could be seen as well from the BFO-formula (1).

As it could be seen from Fig. 3, 7, 11 the company value 'V' increase with level of leverage 'L' at all 'g' values and all company ages n = 3; 6; 9. The company value 'V' increase with company age, as well as with increase of falling rate g. Each triple of curves formed for a company of a fixed age 'n', is ordered as follows (from bottom to top): g = -0.05; -0.04; 0. **The distance between the curves corresponding to different fall rates increases with the age of the company.** This means that influence of falling rate g increases with company age 'n'. For example, under decrease g on 5% (g = -0.05): for nine-year company (n = 9) company value, V, decreases by 12.3%, while for six-year company (n = 6) company value, V, decreases by 10.9%, and for three-year company (n = 3) company value 'V' decreases by 4.7% only. An important conclusion is that the impact of the rate of decline in revenue g on the value of company 'V' increases significantly with the age of company 'n'.

The equity cost, ke, as it is seen from Fig. 4, 5, 8, 9, 12, 13 linearly grows with level of leverage L at all

falling rate g and all company age 'n'. The tilt angle of the curve ke (L) grows with g, but decrease with company age 'n'. There is intermixture of the lines ke (L), corresponding to company ages six and nine years at different falling rate 'g'. It could lead to some interesting effects, because the cost of equity, being an economically justified amount of dividends, determines the company's dividend policy.

We study the dependence of weighted average cost of capital, WACC, on company age 'n' at two values of falling rate, g = -0.04 and g = -0.05, at three values of kd (0.03; 0.04; 0.05) and two values of leverage level L = 0.5 and 0.1. We try to clarify whether "the Golden age" effect [17] takes place at these particular values of equity and debt costs (k0 = 6%; kd = 3%).

We conclude, that the "Golden age" effect is absent at these particular values of equity and debt costs (k0 = 6%; kd = 3%), because WACC decreases with company age, n, monotonically at all three values of kd (0.03; 0.04; 0.05) at all leverage level values (0.5 and 1) and both falling rates g (-0.04 and -0.05).

It was found, that WACC decreases in debt value, kd (0.03; 0.04; 0.05). All three curves WACC(n) tends to the same limit because, as it is seen from the formula (3) WACC does not depends on debt cost value 'kd' in the eternal Modigliani – Miller limit.

The study will allow companies to take into account the behavior of the main financial indicators in the face of declining revenues and will allow reduce risks for companies.

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**V.L. Kulik** — validation, formal analysis, investigation.

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