Review of Business and Economics Studies

REVIEW OF BUSINESS AND ECONOMICS STUDIES (ROBES) is the quarterly peer-reviewed scholarly journal published by the Financial University under the Government of Russian Federation, Moscow. Journal's mission is to provide scientific perspective on wide range of topical economic and business subjects.

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ISSN 2308-944X

Вестник исследований бизнеса и экономики

Издательство Финансового университета
123995, Москва, ГСП-5,
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Тел. 8 (499) 277-28-19.
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Журнал "Review of Business and Economics Studies" ("Вестник исследований бизнеса и экономики") зарегистрирован в Федеральной службе по надзору в сфере связи, информационных технологий и массовых коммуникаций 9 июля 2013 г. Свидетельство о регистрации ПИ № ФС77-54658.

Подписано в печать: **18.12.2015.**
Формат **60 × 84 1/8.**
Заказ № 1058 от 18.12.2015.
Отпечатано в ООП Издательства Финуниверситета (Ленинградский проспект, д. 49).
16+

Review of Business and Economics Studies

Volume 3, Number 4, 2015

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Вестник исследований бизнеса и экономики

№ 4, 2015

СОДЕРЖАНИЕ

Innovation Policy in Contemporary Russia and the Struggle for Influence between the Leading Groups within the Establishment*

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Abstract. The article deals with the issue of choosing the innovative modernization strategy in the Russian Federation and with the suggestions made by the leading groups within the establishment concerning the growth incentives. We examine various scenarios of lobbying the industry and administration interests in the public innovation policy development process. We examine the strengths and weaknesses of the priority innovative development areas suggested by the industry representatives. We make a forecast for a mid-term related to the Russian innovation project transformation, given the "war of sanctions" and the import substitution policy implemented.

Аннотация. В статье рассматривается проблема выбора стратегической линии в ходе инновационной модернизации Российской Федерации и предложения основных элитных групп относительно «драйверов роста». Исследуются различные варианты лоббирования отраслевых и аппаратных интересов в государственной инновационной политике страны. Анализируются сильные и слабые стороны продвигаемых «отраслевиками» приоритетных направлений инновационного развития. Дается среднесрочный прогноз трансформации инновационного проекта России в условиях «войны санкций» и заявленной политики импортозамещения.

Key words: Innovation, modernization, establishment, lobbying, a new industrial policy, state, import substitution.

The Russian modernization project has been lacking a balance and a solid structure since the period of *perestroika*. Attempts were made in 1990–1992 to finalize the project and develop a strategy (cf. *500 Days* program developed by Grigory Yavlinsky, a program developed by Yegor Gaidar, the programs developed by the Civil Union and others.) However, they all have failed because of the poor skills of the reformers, or poor assessment of the political and socio-economic potential, or lack of consistency between the transformational theory and the reality. In addition, the strategy required that the liberal reforms undertaken should be consistent with the local political culture that differed significantly from the culture observed even in the Eastern European countries and the Baltic states, let alone the developed countries of the West, and the underlining Russian ontological theory was not just inconsistent with, but sometimes was in contradiction with those theories. As a result, while facing a political defeat, the 'young reformers' opted for making a number of compromises. The first step was to make the pragmatic ministers (V.Chernomyrdin, G.Khizha, V.Shumeiko) occupy the key positions in the government of the Russian Federation in May 1992. This was followed by the appointment of Chernomyrdin as chairman of the government and making many ideology-blind professional managers members of the cabinet. The same trends were even more pronounced at the regional level, where the 'democrats' were forced to cooperate with the old bureaucracy. At the same time, it is worth mentioning that people fairly quickly grew disappointed with the

* Современная инновационная политика России в контексте борьбы за влияние ведущих элитных групп. Статья подготовлена при финансовой поддержке Российского гуманитарного научного фонда, проекты № 15-02-00080а и № 15-03-00737а.

liberal reforms that were largely rejected by the Russian society due to its political attitudes. The well-known October revolt in Moscow and the outcome of the Duma elections in 1993 and 1995 were the manifestation of the fact. Accordingly, persistence of Boris Yeltsin and his team in implementing the liberal reforms was fraught with political defeat. Thus, in fact, the 'pure' liberal modernization strategic program implemented in the early 1990-s failed. Judging by all appearances, the Russian reformers who were experiencing a cognitive dissonance because the theory contradicted the practice opted for totally abandoning the idea of developing a clear and concise action plan and focused on resolving the tactical issues.

It is noteworthy that the Russian government continued implementing such a policy after Vladimir Putin and his multicomponent team came to power. The team composition was mixed and included the *siloviki* the president worked with when he was a KGB officer in the times of the USSR, the liberal economists he worked with at the St. Petersburg City Hall, seasoned managers who traditionally acted in accordance with the orders made by the government, the young creative class representatives who were attracted by the career building opportunities, patriotic policy-makers who saw Putin as a new hand of iron, and many others. In order to maintain the balance, the government distributed the 'turfs' among the loyal members of political groups, communities and *nomenklatura*.

At the same time, V.Yu.Surkov proposed a 'unifying' national idea, the concept of a sovereign democracy, which was supposed to become an attractive concept for both the patriotic community (because of the 'power doctrine' content) and for the liberal community (because of the 'democratic' content), which was a matter of principle to the community members. Such a disposition was of controversial value as far as the national modernization course was concerned. The strength of the concept was in the fact that the multifaceted Putin's team could guarantee a civil consensus and a relative political stability needed to carry out the necessary political and economic reforms in the army, in the education sector, in youth upbringing and in spiritual life.

However, the 'dispersity' of the Russian establishment hampered a formalized strategy development; otherwise it would have impaired the loyal political and economic establishment groups' interests. However, as long as Russia was in a state of relative calm and the economic and political situation was favorable, a comprehensive top down modernization policy gave positive results. The establishment used their assets to get some interest and, in general, preferred not to be in conflict, but to cooperate. The inflow of petrodollars improved the living standards of a significant part of the population, especially in big cities and metropolitan areas that were traditionally full of people who were in opposition to the government. The reform process was going on with the relative ease in different sectors, and the spontaneous patriotic attitudes were spreading among the people.

Then we observed the overlap of the 2008–2010 crisis, strained relations between Russia and the Western countries over the Russo-Georgian war of August 2008 and the disturbance of balance in the political system as a result of the 'ruling tandem' emergence. All of the above put a number of issues on the agenda. Should Russia continue pursuing the modernization course? What should the modernization be like and what modernization format should be used? Who will be the key driver of the transformations? What are the reforms that Russia needs? Is it possible to carry out the reforms while maintaining the political stability and the status quo within the establishment? The concept of innovation, the concept of Russia as a strong, well-developed and modern state of the 21st century has been introduced into the Russian political discourse.

Vladimir Putin spoke for the first time about the Russian innovative development priorities in his Address to the Federal Assembly delivered on May 10, 2006. He mentioned the following goals:

- To ensure the inflow of investment into the production infrastructure and the innovative development sector. In particular, it was mentioned in the Address that there was a need to build such an environment in the country that could enable turning the production of new innovative knowledge 'into manufacture'[1].
- To promote applied research done at the research centers. The Russian president stated that only in this way the research sector modernization would not be formal, but rather would focus on producing products to be used by the national economy and introducing advanced research products to the market[2].
- To proactively involve the business community representatives in the innovative reforming process.

At the Security Council meeting devoted to the issues of fulfilling the objectives described in the Address of the President of 20 June 2006, V.V.Putin once again mentioned a need to set economic incentives for the businessmen to be come more engaged in the modernization process, thus ensuring that an environment is formed that would be favorable for generating new knowledge and technologies[3].

[1] Address of the President of the Russian Federation to the Federal Assembly, 10 May 2006.
[2] Ibid.
[3] Opening Address by V. Putin at the Security Council meeting dedicated to the issues of fulfilling the objectives described in the Address of the President of 20 June 2006.

D.A.Medvedev, the successor to Vladimir Putin, supported the innovative development trend. He described the Four I (Innovation, Investment, Institutions, Infrastructure) concept[4]. Moreover, in his Presidential Address to the Federal Assembly made on 5 November 2008[5] he made public a list of prioritized goals, which, *inter alia*, included the following:

• To build a personnel poll and attract the most talented, creative and professional staff to positions in government institutions at various levels;

• To revive the best national educational system traditions;

• To produce and export knowledge and advanced technology, to occupy the leading position in the sector of research and education;

• To restructure the public administration system in order to adjust it to the innovative development process;

• To develop an innovative development ideology and innovative development programs for the establishment and the people at large.

According to D.A.Medvedev, Russia should fulfill the objective of building a new economic system that would provide for interaction between its various parts[6].

After Vladimir Putin regained power as president of the country, the innovative modernization machine gained new momentum. He set the objective for the country to occupy the leading position in the world during his election campaign in his major article titled 'We Need a New Economy'. According to him, in order to build an efficient mechanism for modernizing the country's economy, it is necessary to bridge the technological gap between this country and the leading countries of the world. The Russian president said that, as far as the international division of labor is concerned, Russia should position itself not only as a large-scale energy and raw materials supplier, but also as a player on the high-tech product market in a few sectors at least[7]. According to V.Putin, it is necessary for the Russian economy to use a solid legacy of fundamental research and the available Soviet pilot production centers in order to start generating innovations. Accordingly, he promised that the public research foundations supporting the researcher teams' initiatives aimed at developing research products would get 25 billion rubles by 2018. According to Vladimir Putin, we need to get rid of the inertia of major domestic capitalists who got unaccustomed to launching innovative projects, doing the research and making pre-production tests; while 47 state-owned companies have adopted their own innovative development programs, private companies should also get used to the thought that 3% to 5% of their gross income should be used for research and development purposes[8].

However, as soon as the establishment and people at large call for the advanced development, the issue of ideology and reform strategy formation is raised. Despite the fact that the innovation-based modernization is a complex and multidimensional process, almost all of innovation-based modernization models have a strategy at the core of them. There has been no serious disagreement about the modernization plan in the Russian establishment recently. Everybody wants the Russian Federation to become a powerful, developed, modern state and a member of the global leaders' club. It is the strategy development itemized agenda that the establishment started major discussions about, and those discussions were in part caused by the fact that the champions of certain modernization models had particular preferences, and in part by the fact that they had their own selfish interests (industry representatives' interest lobbying, a desire to 'efficiently dispose of' the public funds allocated and so forth.)

In total, one can identify 6 basic theories regarding the innovative development agenda within the Russian establishment.

The first group is composed mainly of the systemic liberals (A.Chubais, A.Dvorkovich and others). It promotes the idea of an innovative breakthrough based on nanotechnology development. Such proposals stand to reason, as the nanotechnology sector is a new 'uncharted land' for all the global political and economic players, and the Russian nanotechnology sector's rapid development will give the country a chance to not only catch up with but also surpass its competitors on the international arena. Accordingly, achieving success in this sector would guarantee big political gains. Firstly, the government would get people's support comparable to the enthusiasm shown by the Soviet people after the flight of Yuri Gagarin into space; secondly, the position of Russia on the international market would become much better. However, the nanotechnology-based modernization project has largely remained a great theory, as no practical implementation stage followed. For example, Rusnano company, despite the enormous public funds allocated to it, failed to develop and 'churn out' a single breakthrough product. In May 2013, the state corporation was criticized harshly at the Accounts

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[4] Dmitry Medvedev's Krasnoyarsk Formula, *Nevskoye Vremya*, 2008, 16 February.

[5] Address of the President of the Russian Federation to the Federal Assembly of 5 November 2008.

[6] Ibid.

[7] V.V. Putin, We Need a New Economy, *Vedomosti*, 2012, 30 January.

[8] Ibid.

Chamber Board meeting. When performing due diligence, the Chamber officials found that the funds allocated to the company had been spent inappropriately and inefficiently. It turned out, for instance, that Rusnano had allocated 47 billion rubles to overseas foreign entities of various kinds while not giving reasons for doing this.

Skolkovo, a domestic version of the Silicon Valley, is another innovative project where priority is given to the nanotechnology development but here, too, the results produced so far are miniscule. In addition, following the audit of the Skolkovo Foundation conducted in the period of April 2013 — August 2013 by the Prosecutor General's Office, violations of law were detected that showed that some of the Skolkovo management's representatives were lacking integrity and were involved in corruption schemes. According to the *Novye Vedomosti* newspaper, about 50 billion rubles were allocated to the Foundation, and the Foundation was absolutely free to use the funds in any manner, as no specific targets were set. 22 billion rubles of the above amount were placed on deposit accounts and used for purchasing promissory notes, the interest on which was used for purposes other than scientific research, which was clear. Moreover, following the fueling tension in relations between Russia and the Western countries, the technology sector's influence upon the government institutions became much weaker as the sector tended to reach a compromise with the US and the EU. It is appropriate to mention here the detainment of two representatives of Anatoly Chubais' nanotechnology expert team by the law-enforcement agencies in July 2015. Leonid Melamed, the former head of Rosnanotech, who had been accused of financial abuse, was among the detained persons.

The champions of the second Russian innovative development model are the Russian government pragmatists who put forward the idea of IT technology development priority. Everybody knows that in today's 'network-based' society the information technology and computer science development are the areas where very promising and even breakthrough products could be created, especially in the Russian Federation, a country with a vast territory and a population that is poorly connected to the data exchange environment. Some steps have been taken recently to improve this. According to *TAdviser* portal, rapid growth was observed on the regional information distribution market in Russia in 2011–2012. A fairly good progress was made within the framework of the E-Government project. In 2011, Russia occupied the 27th place in the world UN E-Government ranking (E-Government Survey 2012: E-Government for the People). It is now closer to the top as it occupied the 59th place in the past ranking. A major involvement of the urban population (not only youth, but also middle-aged citizens) in the data exchange environment was observed. The law 'On Information, Information Technologies and Data Protection' and other laws were adopted in 2006. However, the champions of this theory of innovative development priorities could not make the upper echelons of power recognize their theory in full. It is worth stressing that this 'core' modernization theory has been seriously compromised by the recent political events. In particular, the Arab Spring wave has shown that people's involvement in the data exchange environment is fraught with not only proactive 'conquest of space and time' but also leads to Twitter-aided revolutions that threaten the stability of the state. This is something the Russian government could not be in favor of. However, the senior management of the Ministry of Communications of Russia still thinks about making the IT sector and IT technology the core of the national innovative development project. Ironically, the economic sanctions imposed by the West against Russia in the summer of 2014 played into the hands of this modernization theory. In such unfavorable conditions the Russian government ordered that the imports be substituted in as many sectors as possible, including the IT and communication sector. In August 2014, Nikolai Nikiforov, Minister of Communications quickly acted in line with the order given at the top. He told the reporters that measures were taken in Russia that would allow import substitution in the software sector within the period of three to seven years. According to the Minister, they develop a comprehensive program to support software development in a whole big sector year after year while progressing step by step in order to substitute the imports, they develop a mechanism that will allow them to strengthen and support the industry, although it will not be a quick process, it will take three or five or seven years in some areas. However, he did not specify which products he referred to but said the country depended on imports of many types of software, including mobile operational systems, database management systems. He said that non-budgetary funds would be used for those purposes. N.Nikiforov said that the program should not be financed using taxpayers' money, there should be a mechanism developed in the industry that would help resolve this major issue.

The third innovative modernization theory is championed by part of the Russian establishment (i. e. the Russian Railways company management and senior officers of the Ministry of Transport). The theory envisages the transport arteries development. It is assumed that the strategic position of Russia between

Europe and Asia can become both a serious geopolitical and economic advantage. The theory champions are in favor of rapid construction and upgrade of strategic roads, railways, and air transit hubs. This means the project champions want Russia to become the 'moderator of space and time' on the Eurasian continent. In general, the transport and communications development was supported by the Kremlin officials and the general public.

It is noteworthy that not long ago, in June 2014, Dmitry Medvedev, chairman of the Russian government, signed a decree that approved a new version of the Transport Strategy of the Russian Federation for the period of up to 2030. It is aimed at developing a single Russian transport area that is based on a balanced advanced efficient transport infrastructure development, thus ensuring transportation services quality and public access to transportation services in accordance with social standards, ensuring access to and quality of transportation and logistics services in the freight transportation sector, integration into the world transportation services market, use of the country's potential in the transit services sector, and the transport system safety improvement. All of the above is stated in the explanatory note attached to the document. Amendments have been made to the timeframes and the stages within certain projects, including the high-speed railway development projects; an action plan has been updated for the Medium Term Transport Strategy Development Program (2014–2018). A draft of the decree was developed by the Ministry of Transport of Russia to implement the resolutions adopted at the government meeting on August 27, 2013.

However, there are some shortcomings in the above-mentioned project. Firstly, the political and economic situation is unfavorable that followed the Ukrainian crisis and the sanctions against Russia imposed by the US and the EU. Secondly, this large-scale project is a very costly one. It requires huge government investments, and the Russian government that has to deal with the sanctions tries not to become engaged in big risky spending schemes. Thirdly, the establishment and the general public are the champions of a controversial theory that says that the power of Russia is in the 'weakness' of its roads. They prove their paradoxical point by citing the fact that the German war machine 'slipped' on the Russian territory in the fall and winter of 1941.

The fourth innovation theory champions are the generals from the military industrial complex. They want the public to embrace that fact that the military industrial complex has traditionally been a generator of new ideas and technologies, and that this sector, unlike most other sectors that were seriously damaged in the 1990-s, maintains the industrial infrastructure, keeps the personnel and has the funds that are used to finance the R&D sector.

In August 2014, Dmitry Rogozin, deputy chairman of the Russian government, published an article in the Natsionalnaya Oborona journal. He argued that military industrial complex should become the locomotive of the economy development. In particular, he has noted that the current situation in Russia is very similar to the one that was observed in the late 1930-s when the Soviet Union was forced to become engaged in the rapid industrialization in order to defend itself, while being in economic and political isolation. Therefore, the military industrial complex should become the locomotive for the modern Russia development, just as it has been during the last 7 decades.

Yet, not only Dmitry Rogozin, deputy chairman of the Russian government who supervises the military industry operations, but also a few other 'heavyweights' (Union of Mechanical Engineers, Russian Technologies State Corporation, etc.) share this view. For instance, Yuri Koptev, chairman of the Scientific and Technical Council of Russian Technologies State Corporation, has recalled more than once that president Vladimir Putin has set the goal to profoundly diversify and improve the technical infrastructure and technology within the defense industry as it is the locomotive of our economy. According to him, in the Soviet times, a lot of innovations that were coming from the military industrial complex were introduced into the civilian life, the complex provided products for the society to use. Aleksandr Ageyev, director of the Institute for Economic Strategies of the Russian Academy of Sciences, also believes that the economy development is not possible without the military industrial complex modernization. According to him, the military industrial complex is at the core of the economy, and technology development, employment, and security depend on it. This is why the Russian military industrial complex has traditionally played the role of a locomotive in the economy development process. Moreover, the military industrial complex depends very much on the goal-setting done by the government, so in an economy based on private property in which private traders are only interested in getting profit, the military industrial complex can become an instrument of economic modernization not only due to the military products supply. One needs to understand that the economic development issue is an issue whose resolution depends on a number of factors, i.e. ruble stabilization, financial policy, and so on. The WWI is a war that is based on the land-based weaponry use; the WWII is a war where the flying motors and artillery were used; the war of the future is associated with the use of robots, drones, genetic systems, new materials. This is why the military industrial complex should be modernized, and the economy modernization process will run in parallel.

Given the fact that the news on the fundamentally innovative and advanced weapons development within the framework of the military industrial complex has been increasingly circulating in recent years, military lobbyists' voices begin sounding increasingly confident. It is noteworthy that the defense industry representatives often go beyond civilian researchers as far as innovative project development is concerned. The so-called exoskeleton, a special suit with embedded special devices, that allows a person to carry extra load, can be an example of this. Skolkovo center representatives informed the public about the innovation in August 2014 while getting as much media coverage as possible when showing the exoskeleton that could make life of disabled persons easier. At the same time, it is well-known that such products developed for military use have been available to the military industrial complex for a long time, and it is probable that they were produced on a large scale.

Moreover, there are two factors that play into the hands of the 'hawkish' innovative development scenario champions. Firstly, sharp aggravation of relations between Russia and the US and the EU, with the countries being on the verge of a new Cold War, calls for an appropriate Russian Army supply of precision weapons and other advanced products. Secondly, the official data on the growth of military technical cooperation between Russia and other countries of the world are impressive. The unofficial data might be even more impressive. For example, Russian weapons sales on the foreign markets totaled USD 15.7 billion in 2013. Thus, according to pro-defense industry activists, the defense industry development will let the country obtain a lot of funds immediately. However, the liberal groups within the establishment are pronouncedly against the 'hawks'. They fear the militarization of the country and do not want the relations between Russia and the West to become even more aggravated. In addition, despite all the patriotic attitudes, the Russian people still have that fear of becoming poor that they had in the 1980-s. At that period, a sharp increase in military spending undermined the social and economic stability in the Soviet Union and led to civil industries' degradation.

Those who support the idea of creating incentives for the military industrial complex development go hand in hand with the new industrial policy champions who are also in favor of improving the real sector financing. They believe that not all the global leaders make a bet on the breakthroughs of the postmodern period. Many achieve the goal of advanced development by using existing industrial capacity in a new way and by introducing new technologies. New industrial policy champions indicate that the Russian industrial potential degraded seriously and was ruined in the 1990-s. This led to the loss of the economic position on the international arena in a period following a systemic crisis. According to them, the political and economic renaissance of the 2000-s was largely due to the extensive use of oil and gas, while the industrial enterprises that had survived during the period of reforms of the 1990-s, continued struggling without the strong public support.

At the same time, after getting solid financial investment, the country's industry could make a major innovation-based breakthrough. The import substitution campaign that followed the introduction of Western sanctions against Russia let the domestic producers become more optimistic. In August 2014 high-class projects were launched that supported industrial development programs that were associated with both heavy and light industry development. However, this project has its shortcomings too. Firstly, full recreation of potential in the Russian production sector is a very ambitious goal, if we mean ensuring self-sufficiency.

In addition to the substantial financial investment issue, there is yet another issue the Russian government is thinking about. It has doubts as to the funds use efficiency, if funds are ever allocated. After all, the corruption factor plays virtually the most important part in the Russian economy, and at least half the amount of the entire project financing is misappropriated because of this factor's impact. At the same time, the industrial development theory champions have repeatedly tried to use the funds from the Stabilization Fund and the Reserve Fund of the Russian Federation using the pretext of production sector development requirements, which led to acquiring serious enemies among the management of the Ministry of Finance and even the Ministry of Economic Development.

In addition, there are strong doubts about the Russian industries' competitiveness even following the product quality improvement. The leading Western countries are far ahead of us in technology development in some areas (especially non-military development sectors) and the third world countries have an opportunity to lower the prices significantly because of the availability of cheap labor force. The choice of the new industrial policy as a strategic innovative development trend will largely depend on the external environment and on the degree of coherency of the public import substitution policy. In any case, pro-industrial policy activists will manage to get funds for innovative development financing, but it is unlikely that the domestic civil industry would become a driver of growth.

In addition to those listed above, there is yet another national innovative breakthrough project. All bets in it are put on the nuclear industry development. It is clear that Rosatom management and Sergei Kiriyenko, head of Rosatom, are the key champions of this theory. The theory key point is that the nuclear industry, while being a special strategic industry, not only maintained its strong potential preserved in the Soviet era, but also improved it during the 1990-s and the 2000-s. Moreover, the sector representatives managed to get profit from entering the foreign markets and acquired reliable and creditworthy customers who were often the political enemies. For instance, Rosatom cooperates with the Islamic Republic of Iran and the United States demonstrating the same degree of productivity. It is also worth mentioning that the nuclear industry uses powerful technology that not just meets the international standards but surpasses them in certain areas.

At the same time, this project is controversial to a large extent. The 'Chernobyl complex' is still observed in the post-Soviet states and in Europe, and the public demonstrates absolute disapproval when it comes to the projects associated with posing a threat to the environment. It is difficult for the innovative project managers to get Russian government' approval because of some HR decisions made. Sergei Kiriyenko is largely perceived as an 'alien' by the Putin's 'St. Petersburg team' members. As a consequence, his agency's projects are unlikely to be considered a priority. In addition, the head of Rosatom has a reputation of being a liberal and a scientologist, which is of dubious value, given the presence of patriotic enthusiasm and the dominant ideology of 'empire values', which provides a major advantage to Kiriyenko's government opponents.

Thus, we can observe that a pointed debate about the major trends in innovative modernization policy is still going on in Russia while often reflecting the selfish lobbyists' interests. The president of the Russian Federation prefers to sit on the fence and abstain from taking a final decision in favor of some group of lobbyists in order to maintain the unity of the 'ruling class'. Anyway, given the sharp aggravation of relations between Russia and the West, the decision as to what the priorities should be in the innovation-based political and economic modernization will have to be made, and, under such galvanizing circumstances, the pro-defense and nuclear industry activists have the best chance to win, as far as the strategic acceptance of their theory is concerned.

REFERENCES

Seleznev P.S.*Innovatsionnye proekty sovremennosti: politiko-ekonomicheskii opyt dlia Rossii* [Innovative projects of modernity: the political and economic experience for Russia]. Moscow: Rossiiskaia politicheskaia entsiklopediia (ROSSPEN), 2013. 302 p.

Seleznev P.S.*Gosudarstvennaia innovatsionnaia politika stran Zapada i Rossii (konets XX — nachalo XXI veka)* [The state innovation policy of the West and Russia (the end of XX — the beginning of XXI century]. Moscow: Finansovyi universitet, 2012, 204 p.

Makarov V.L.*Gorizonty innovatsionnoi ekonomiki v Rossii. Pravo, instituty, modeli* [Horizons of innovation economy in Russia: Law, institutes, models]. St. Petersburg: Lenand, 2010. 240 p.

Druker P.F.*Biznes i innovatsii* [Business and Innovation]. Moscow: Vil'iams, 2007. 293 p.

Emel'ianov, Iu.S.*Gosudarstvenno-chastnoe partnerstvo v innovatsionnoi sfere: zarubezhnyi i rossiiskii opyt* [Public-private partnership in innovation: foreign and Russian experience]. Moscow: Librokom. 2012. 253 p.

Innovatsionnaia politika i regional'noe razvitie v sovremennom mire [Innovation policy and regional development in the modern world]. Moscow, INION RAN. 2011. 198 p.

Innovatsionnaia politika. Rossiia i Mir. 2002–2010 [Innovation policy. Russia and the World. 2002–2010]. Moscow: Nauka. 2011. 456 p.

Shatilov A.B.*Innovatsionnoe razvitie Rossiiskoi Federatsii i interesy vedushchikh elitnykh soobshchestv* [The innovative development of the Russian Federation and the interests of the leading elite communities]. *Regional'nye problemy preobrazovaniia ekonomiki (Makhachkala) — Regional problems of economic transformation (Makhachkala)*, 2014, no. 8 (46), pp. 14–20.

Shatilov A.B.*Innovatsionnyi proekt modernizatsii Rossii i pozitsiia vlastnoi elity* [An innovative project to modernize Russia and the position of the ruling elite]. *Vlast' — Power,* 2013, no. 9, pp. 16–20.

Shatilov A.B.An innovative project in Russia and the creative class: // contemporary debates, *Elitologiia Rossii: sovremennoe sostoianie i perspektivy razvitiia. Materialy Pervogo Vserossiiskogo elitologicheskogo kongressa s mezhdunarodnym uchastiem* [Elitologii Russia: current state and prospects of development. Materials First All-Russian Congress]. Rostov-on-Don: IuRIF RANKhiG, 2013. T. 1. Pp. 427–434.

Energy Price Shocks in Dynamic Stochastic General Equilibrium: The Case of Bangladesh[1*]

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Abstract. We investigate the role of energy price shocks on business cycle fluctuations in Bangladesh. In doing so, we calibrate a Dynamic Stochastic General Equilibrium (DSGE) model, allowing for both energy consumption by households and as an input in production. We find that qualitatively temporary energy price shocks and technology shocks produce similar impulse response functions, as well as similar (quantitatively) auto-correlations in aggregate quantities. The variance in aggregate quantities are better explained by technology shocks than by energy price shocks, suggesting that technology shocks are more important source of fluctuations in Bangladesh.

Аннотация. Мы исследуем влияние колебаний цен на электроэнергию на флуктуации бизнес-цикла на примере Бангладеш. В этом исследовании мы калибрируем динамическую стохастическую модель общего равновесия (DSGE-модель), учитывающую бытовое и промышленное потребление электроэнергии. Мы пришли к выводу, что временные колебания цен на электроэнергию и колебания производительности приводят к схожим ответным реакциям, а также к количественно схожим автокорреляциям суммарного количества. Расхождение в суммарных количествах лучше объясняются технологическими колебаниями, чем колебаниями цен на электроэнергию. Это приводит к выводу, что технологические колебания являются более важным источником флуктуаций бизнес-цикла в Бангладеш.

Key words: Energy price shocks, business cycles, dynamic stochastic general equilibrium.

1. INTRODUCTION

Standard Dynamic Stochastic General Equilibrium (DSGE) models typically assume that exogenous technology shocks identified through the Solow residual are the main sources of aggregate fluctuations in the economy. This concept has often been criticised as in De Miguel *et al*. (2003). They argue that there is a lack of discussion on the nature of technology shocks, which are unobservable, and based on the idea that they are just the result of the convergence of other kinds of factors that are not specified in the model. One of the identifiable sources of shocks that have claimed the attention of many economists is energy price shocks which, according to some researchers, being equivalent to adverse technology shocks can induce significant contractions in economic activity. In fact, using US data, Hall (1988) finds that a standard measure of technology, the Solow residual, systematically tends to fall whenever energy price increases. The case for incorporating energy price shocks into the DSGE models has subsequently been made credibly by McCallum (1989).

Authors such as Kim and Loungani (1992), Finn (2000), Rotemberg and Woodford (1998), Dhawan and Jeske (2007), De Miguel *et al*., (2003, 2005), Tan (2012) investigate the effect of energy price shocks on the variation of output using the DSGE framework. Most of the authors find that such energy price shocks offer very little help in explaining the US business cycle, therefore supporting the views of macroeconomists who downplay the impact of energy price shocks on

* Колебания цен на электроэнергию в динамической стохастической модели общего равновесия на примере Бангладеш.

[1] We wish to thank Parantap Basu, Angus Chu, Guido Cozzi, Tooraj Jamasb, Peter Sinclair, two anonymous referees and the Executive Editor Alexander Kaffka for helpful comments which greatly improved the paper.

the economy. For instance, Tobin (1980) has argued that the share of energy in US GDP is so small that it would require implausible parameter values to generate strong aggregate impacts from energy price shocks.

Although the above researchers investigated the theoretical relationship between energy and macroeconomy through different possible channels, upon closer analysis, two common characteristics can be seen for most of the aforementioned models. Firstly, energy is considered primarily in the production function, overshadowing its importance in the household's utility function. Secondly, all the models are found to be calibrated to reflect the scenarios of developed countries, mainly US economy leaving open the question of whether energy price shocks can explain macroeconomic fluctuations in developing countries.

This papers aim at filling the above gaps in the literature by providing a framework to analyse the relative impact of energy price shocks and technology shocks for Bangladesh. To the best of our knowledge, there is yet no record of an energy augmented DSGE model which has been calibrated for developing economy to investigate the interactions between energy and the overall economy. Differently from the above models on energy price shocks, we include energy both in the utility and production function, to recognise the importance of energy for household's welfare, which is particularly relevant for developing countries (Jamasb, 2006). Our model therefore constitutes a useful benchmark framework to address the behaviour of different macroeconomic variables for policy analysis in developing countries.

In particular, we first calibrate our DSGE model to explain the quantitative properties of macroeconomic variables for the Bangladesh's economy. Then we examine how the fluctuations of key economic variables such as consumption and output are explained by the exogenous shocks. The model's ability to describe the dynamic structure of the Bangladesh economy is analysed by means of the Impulse Response Function (IRF) which yield useful qualitative and quantitative information.

Our results show that the basic DSGE model can replicate some of the main features of the Bangladesh economy for the period 1990–2010. In addition, we demonstrate that energy price shock is not the main explanatory factor of the macroeconomic fluctuations in Bangladesh. Consequently, we conclude that output fluctuations in Bangladesh are mainly driven by technology shock. Our results further reveal that the exogenous shock's impact on endogenous system variables are in the right direction.

The paper is organised as follows. The model is depicted in section 2 followed by a discussion on calibration of the parameters in section 3. Section 4 portrays the analysis of the results obtained and finally, in the last section, we present the conclusions.

2. THE MODEL

We assume a representative agent model where all economic agents are identical and act as both a household and a firm. Energy is explicitly modelled in the household's utility function where the representative household derives utility from the consumption of energy, from standard consumption, and from leisure. Following Finn (2000), we measure energy oriented goods as the sum of electricity, coal, natural gas and petroleum. Standard consumptions include all the durable and non-durable goods excluding energy goods. Each household's endowment of time is normalised to 1 so that leisure is equal to $(1-l)$ where l represents the number of working hours.

Household consumes a Constant Elasticity of Substitution (CES) aggregation of energy and standard consumption, and also derives utility from leisure. Thus for the household, in each period it decides on how much energy goods to consume (e_t), how much to consume of the standard consumption good (c_t) and how much time to devote to labour (l_t) in order to maximise its lifetime expected utility[2].

$$\max \mathrm{E}_0\left(\sum_{t=0}^{\infty}\beta^t u_t\right)$$

With a per-period utility function of the following form:

$$u_t = \varphi \ln[\theta c_t^{\rho} + (1-\theta)e_t^{\rho}]^{\frac{1}{\rho}} + (1-\varphi)\ln(1-l_t) \qquad (1)$$

The utility function exhibits the commonly assumed properties like $u_c > 0$, $u_{cc} > 0$, $\lim_{C\to 0} = \infty$ and $\lim_{C\to\infty} = 0$. That means, additional consumption and leisure increases utility but does so at a diminishing rate.

Here, φ represents the share of consumption in the household's utility where $\varphi \in (0, 1)$. θ is the share of standard consumption in the household's aggregator where $\theta \in (0, 1)$. With this aggregation function, the elasticity of substitution between energy and standard consumption is $\sigma = 1/1-\rho$. When $\rho = 0$ and $\sigma = 1$, the CES function becomes Cobb Douglas (CD) function. It is rational to choose $\rho < 0$, which implies that the goods are somewhat complementary.

[2] Due to the shocks, which follow a known probability distribution, future consumption, leisure, etc are uncertain, so we adopt expected utility as the objective function for the household.

Following Kim and Loungani (1992), the production technology of firm is described by a Cobb-Douglas production function, combining energy as an additional input along with capital and labour.

$$Y_t = A k_t^{\alpha} l_t^{\gamma} g_t^{1-\alpha-\gamma} \tag{2}$$

Where α and γ is the fraction of aggregate output that goes to the capital input (k_t) and labour input (l_t) respectively, and $1-\alpha-\gamma$ is the fraction that goes to the energy input (g_t). That means all the economic agents rely on energy either for household's consumption or for production of various goods. Furthermore, energy price is modelled as an exogenous random process in addition to technology shock.

Just as in Cooley and Prescott (1995), the stochastic technology A_t is assumed to follow:

$lnA_t = \omega \ln A_{t-1} + u_t$; where $u_t \sim N(0, \sigma^2)$.

The capital stock depreciates at the rate δ (with $0 < \delta < 1$) and the household invests a fraction of income in the capital stock in each period. So, capital accumulates according to law of motion:

$$k_{t+1} = (1-\delta)k_t + i_t \tag{3}$$

The price of energy used in the economy, P_t, is exogenously given and follows AR (1) process: $lnP_t = \Psi \ln P_{t-1} + v_t$; where v_t is normally distributed with standard deviation τ and zero mean. As energy is consumed both by the consumers and the producers in this model, the economy's resource constraint for period t is given by:

$$Y_t = c_t + i_t + P_t(e_t + g_t) \tag{4}$$

The Lagrangian to the planning problem can be written as follows[3]:

$$L = \sum_{t=0}^{\infty} \beta^t \left(\varphi \log\, [\theta c_t^{\rho} + (1-\theta) e_t^{\rho}]^{\frac{1}{\rho}} + (1-\varphi)\log(1-l_t) \right) + \lambda_t [A k_t^{\alpha} l_t^{\gamma} g_t^{1-\alpha-\gamma} + (1-\delta)k_t - c_t - P_t\,(e_t + g_t)] \tag{5}$$

where λ_t is the Lagrange multiplier and the function is maximised with respect to c_t, k_{t+1}, e_t, l_t, g_t and λ_t.

The first-order conditions are:

$$\frac{c_{t+1}}{c_t} = \beta.[A\alpha\, K_{t+1}^{\alpha-1} l_{t+1}^{\gamma} g_{t+1}^{1-\alpha-\gamma} + (1-\delta)] \frac{1 + \left(\frac{\theta}{1-\theta}\right)^{\frac{1}{\rho-1}} . P_t^{\frac{\rho}{\rho-1}}}{1 + (\frac{\theta}{1-\theta})^{\frac{1}{\rho-1}} . P_{t+1}^{\frac{\rho}{\rho-1}}} \tag{6}$$

$$\frac{c_t}{1-l_t} = \frac{\varphi}{1-\varphi} . \frac{1}{1 + (\frac{\theta}{1-\theta})^{\frac{1}{\rho-1}} . P_t^{\frac{\rho}{\rho-1}}} . [A\, K_t^{\alpha} \gamma\, l_t^{\gamma-1} g_t^{1-\alpha-\gamma}] \tag{7}$$

$$\frac{e_t}{c_t} = (P_t . \frac{\theta}{1-\theta})^{\frac{1}{\rho-1}} \tag{8}$$

$$P_t = A k_t^{\alpha} l_t^{\gamma} (1-\alpha-\gamma) g_t^{-(\alpha+\gamma)} \tag{9}$$

[3] Notice that we could equally well have formulated a competitive economy, where the household faces a budget constraint, taking prices as given, and a representative firm maximizing profits, also taking prices as given. The solution to the planning problem coincides with the competitive equilibrium, i.e. the First Welfare Theorem applies. For computational reasons we choose the planning formulation, as it yields fewer equations to solve.

$$c_t + k_{t+1} + P_t\ (e_t + g_t) = A\,k_t^{\alpha} l_t^{\gamma} g_t^{1-\alpha-\gamma} + (1-\delta)\,k_t \qquad (10)$$

$$Y_t = A\,k_t^{\alpha} l_t^{\gamma} g_t^{1-\alpha-\gamma} \qquad (11)$$

$$lnA_t = \omega lnA_{t-1} + u_t \qquad (12)$$

$$lnP_t = \Psi \ln P_{t-1} + v_t \qquad (13)$$

3. CALIBRATION

Before examining the model's performance to evaluate the empirical data, model calibration is required. In this section, we use the term calibration for the process by which researchers choose the parameters of their DSGE model from various sources. For example, Cooley and Prescott (1995) calibrate their model by choosing parameter values that are consistent with long run historical averages and microeconomic evidence. Dhawan and Jeske (2007) calibrate parameters to produce theoretical moments of model aggregates that reproduce, as best possible, the empirical moments obtained from the empirical data.

However, we have generally adopted three approaches in terms of calibrating parameters for our DSGE model. Some of the parameters are picked from the existing DSGE literature for developing and developed countries (Choudhary and Pasha, 2013). Some of the parameter values are chosen by using steady state conditions of the model. Rest of the parameter values are directly considered from Bangladesh Bureau of Statistics (2015) and Bangladesh Household Income and Expenditure Survey (2015). Due to data constraints, all parameters in our model are calibrated for annual frequency.

There are 11 parameters in total with 7 structural and 4 shock related parameters in the model. Structural parameters can be categorised into utility and production function related parameters. It is important to have a good understanding of rationale behind picking different parameter values in order to properly evaluate the fit of the model. Let us briefly describe our procedure for selecting parameter values listed in Table 1.

First of all, we discuss parameters related to production. Alpha (α), Gamma (γ) and Depreciation (δ) are the main parameters related to production. Following Rahman and Yusuf (2010), we set alpha equals to 0.31 which implies capital's share of national income in Bangladesh is slightly less than a third. This is fairly close to the computed aggregate capital share which is 0.36 as calculated by Tan (2012). However, the average of capital shares of other developing countries is around 0.45 as reported by Liu (2008). According to Bangladesh Household Income and Expenditure Survey (2010), the labour share of output in Bangladesh varies from 0.65 to 0.70. We decided to use a value of 0.65 to make it consistent with the CD production function used in our model. Finn (2000) also mentions that the measures of labour's output share range from 0.64 (Prescott, 1986) to 0.76 (Lucas, 1990).

Table 1. Parameters of the economy.

β, discount factor	0.88
α, capital share of output in the production function	0.31
γ, labour share of output in the production function	0.65
δ, depreciation rate	0.025
φ, the share of consumption in the household's utility	0.41
θ, the share of standard consumption	0.8
σ, the CES parameter of household's utility function	–0.11
ω, persistence coefficient of technology shock	0.95
Ψ, persistence coefficient of energy shock	0.95
ζ, standard error of technology shock	0.01
τ, standard error of energy shock	0.01

Source: Bangladesh Household Income and Expenditure Survey (2015), Bangladesh Bureau of Statistics (BBS, 2015).

Depreciation rate is usually very low in the developing countries. Thus, depreciation rate, δ has been set at 0.025 implying that the overall depreciation rate in Bangladesh is 2.5% annually. This value is equally realistic from the perspective of the developing country's economic condition (IMF, 2001 and Yisheng, 2006). The capital output ratio in Bangladesh is borrowed from Rahman and Rahman (2002) who estimated that the trend in capital output ratio in Bangladesh over the period of 1980/81 to 2000/01 is equal to 2.

Now, we discuss parameters related to household utility. Given, α, δ, capital-output ratio and considering the value of steady state level of price is $P = 1$ (mean zero in the log implies a mean of unity in the level), the value of discount factor beta, is obtained from equations (6) and (11) evaluated in steady state:

$$\beta = \frac{1}{\alpha \frac{Y}{k} + (1 - \delta)}$$

Our estimated value 0.88 is less compatible with the value of discount factor used in other existing literature for developing countries at annual frequency. Ahmad *et al.*, (2012) estimate the long run discount factor for a group of developed and developing countries and find that the discount factor of most of the developing countries is relatively similar to that of developed countries. For example, they calculate the discount factor, β, equals to 0.94 for Philippines. As a robustness check, we have performed sensitivity analysis along three different discount parameters ($\beta = 0.88$, $\beta = 0.96$ and $\beta = 0.99$) and confirm that our results are robust to a wide range of possible β values (see Table 2). It is worth noting from Table 2 that the steady state value of *c* shows odd pattern with low β values. In principle, lower β value should imply a lower level of steady state consumption (as the household is more impatient). However, in this sensitivity analysis, we have also changed the value of δ which offset the changes observed in c for different β values. Thus, lower β value yields a higher value for c in our analysis. However, we have also run another sensitivity analysis keeping the value of δ to 0.025. Our results show that *c* is now smaller for lower β values.

Due to unavailability of the data of working hours, we set $l = 0.33$ with an assumption that people work about one-third of their time endowment which is a widely accepted value for DSGE analysis. For example, *l* is set equal to 0.30, consistent with the time-allocation measurements of Ghez and Becker (1975) for the US economy.

Certain standard parameters are calibrated following standard literature. The share of standard consumption, θ, is set at 0.8. In this paper, the household's utility function follows a general CES form, meaning that it cannot be used to model an elasticity of substitution of exactly 1. Here, it is set at 0.9 for the main analyses, and the CES parameter of the household's utility function, ρ, is therefore –0.11 (1- (1/0.9)), which is negative and indicates that energy and standard consumption are somewhat complementary.

φ reflects the share of energy consumption and standard consumption goods in the household's utility function and its value is found to be 0.41 as follows:

For optimality, the labour-leisure trade off should be such that the marginal rate-of-substitution between leisure and consumption must equal the marginal product of labour (the implied normalised wage rate in the corresponding competitive equilibrium). That means,

$$\frac{U_l}{U_c} = F_l$$

Table 2. Sensitivity analysis for β.

Variables	β = 0.88 and δ = 0.025	β = 0.96 and δ = 0.12	β = 0.99 and δ = 0.14
k	0.712689	0.820228	0.963403
Y	0.370975	0.427755	0.466477
A	1	1	1
c	0.262911	0.242628	0.24319
l	0.331236	0.382276	0.402381
P	1	1	1
i	0.0178172	0.0984273	0.134876
e	0.0754072	0.0695897	0.069751
g	0.014839	0.0171102	0.0186591

$$\frac{\dfrac{1-\varphi}{1-l_t}}{\dfrac{\varphi}{\rho}\cdot\dfrac{\rho\theta c_t^{\rho-1}}{\theta c_t^{\rho}+(1-\theta)e_t^{\rho}}}=[A\,K_t^{\alpha}\gamma\, l_t^{\gamma-1}g_t^{1-\alpha-\gamma}]$$

$$\frac{\dfrac{1-\varphi}{1-l_t}}{\dfrac{\varphi}{\rho}\cdot\dfrac{\rho\theta c_t^{\rho-1}}{\theta c_t^{\rho}+(1-\theta)e_t^{\rho}}}=[\gamma\frac{Y}{l}]$$

$$\frac{1-\varphi}{\varphi}\cdot\frac{l}{1-l_t}\left[1+\frac{(1-\theta)}{\theta}(\frac{e_t}{c_t})^{\rho}\right]=\gamma\frac{Y}{l}$$

By using equation (8), we can calculate the steady state ratio of energy to standard consumption which yields a value of 0.28. Now, given the value of l, γ, θ and the ratio of $\frac{c}{y}$ and $\frac{e}{c}$, we can find the value of φ equals to 0.41.

Owing to the unavailability of data, following King, Plosser and Rebelo (1988), we set the persistence of our two exogenous shocks equal to 0.95 and standard deviation of the shocks equal to 0.01. Using different series, empirical literature gets a range of estimates for persistence 0.85–0.95 and standard deviation 0.0095–0.01.

We assume that the natural log of the technology variable and the energy price follow an AR (1) process, where the shocks are iid with zero mean and variances σ_u^2 and σ_v^2, respectively. Zero mean implies steady state levels $A = 1$ and $P = 1$.

4. RESULTS

After calibration, to evaluate the performance of our model, we compare steady state ratios from the models with their empirical counterpart. Furthermore, second order moments (such as standard deviation, contemporaneous correlation with output etc.) obtained from simulations will also be evaluated from our models and their fit with the actual data[4].

Our model shows that the relevant capital output ratio is equal to 1.92 which is fairly close to the actual data of 2 as explained in the previous section. Another important ratio of our model is the consumption-output ratio. The model does a good job at matching the model generated ratio of 0.70 to the actual consumption output ratio of 0.65–0.70 as showed in data. However, our model undershoots the value of investment output ratio by a large extent. The model-generated result 4.8% is far away from the average long run investment output ratio of 20%.

We would also like to verify the ability of the model to reproduce other empirical regularities of the Bangladesh business cycle. In order to do so, we proceed to the stochastic simulation of the model with the parameters obtained in the calibration section, where the sources of fluctuations come from the technology shock and energy price shock. Table 3 reports a selection of second moment properties for the HP filtered series corresponding to the Bangladesh data and the simulated economy respectively[5]. In other words, we would like to evaluate our model's performance by comparing the results with data. For this purpose, the following table reports some selected historical moments from data and their counterparts predicted by our models.

Our model performs well to capture the actual volatility of output and investment when we consider both the technology and energy price shocks together as well as when we take into account the technology shocks alone. However, considering only energy price shocks is not sufficient. A shock to the energy sector or a policy pertaining to that sector should have significant impact on the rest of the economy. Yet, energy price shocks can account for only 3.29% of output volatility whereas technology shocks can account for almost 83.52% of output volatility in our model. Investment also follows more or less the same pattern as output. Moreover, the model does a poor job in replicating the variation of consumption of energy and non-energy goods. The situation is more severe in the standard consumption when we just consider energy price shocks. Therefore, energy price shocks are a less important source of aggregate fluctuations in Bangladesh economy. Our results reveal from the long run data that energy input is well substituted by other inputs (capital and labour) in the production function when there is any shock in energy price. In fact, the results indicate that there are some mechanisms by which macroeconomic variables could be stable in spite of a limited source of energy inputs as argued by Bartleet and Goulder (2010). Additionally, our DSGE model shows that the series are not strongly persistent and robust in the sense of having a large first order autocorrelation coefficient and matching the historical data. The highest persistent series is

[4] Dynare, a preprocessor and a collection of MATLAB routines is used in this paper to solve for the steady states, linearise the necessary conditions around steady states, compute the moments and calculate the impulse response paths once the necessary equations are transformed into Dynare codes (Griffoli, 2011).

[5] We have used HP filtering data to make it consistent with Dynare generated data as it gives HP filtering data. However, considering the fact that HP filtering data might give rise to spurious cycles as criticised in some literature, we have also checked with Baxter and King (BK) filtering process but that does not make any significant differences.

Table 3. Actual and predicted moments.

	Data*	DSGE Model		
Statistics	**Estimate**	**Model 1 Technology and Energy Price Shocks**	**Model 2 Technology Shocks**	**Model 3 Energy Price Shocks**
Standard Deviation				
Y	0.005488	0.004321	0.004335	0.000172
i	0.003155	0.002264	0.002270	0.000088
c	0.007593	0.001629	0.001637	0.000115
e	0.002546	0.000784	0.000470	0.000624
Standard Deviation Relative to Output				
i	0.57	0.49	0.52	0.51
c	1.38	0.38	0.38	0.67
e	0.46	0.18	0.11	3.62
Autocorrelation				
Y	0.823	0.4815	0.4845	0.4841
i	0.824	0.4406	0.4437	0.4437
c	0.821	0.5777	0.5811	0.5230
e	0.821	0.4879	0.5811	0.4731
Correlation with Output (Y)				
i	0.9965	0.9545	0.9545	0.9550
c	0.9938	0.9457	0.9470	0.9890
e	0.9967	0.5238	0.9470	0.9986

* The statistics are based on log-differenced and HP filtered for the period 1990–2010 to reflect the actual growth rates.

capital which is 0.74 whereas the autocorrelation of the remaining series are typically in the neighborhood of 0.45 compared to their empirical counterpart of a range around 0.82.[6] The policy and transition function reveals that the exogenous shock's impacts on endogenous variables are in the right direction. Lastly, the model captures the fact that most of the series are quite pro-cyclical with output.

After considering the steady state ratios and second order moments for our model with their empirical counterparts, finally we take a brief look at the IRF generated in response to the technology and energy price shocks.

4.1 TRANSMISSION MECHANISMS OF ENERGY PRICE SHOCKS

In this section, we describe the dynamic mechanism in which energy price shock is propagated. The shock is equal in size to the standard deviation of the normalised price. Figure 1 shows the response of the different endogenous variables of the model in presence to such a shock. When there is an increase in relative energy price (P), both the amount of energy consumption (e) and the amount of energy used (g) in the production decreases by 8% and 1.5% respectively. Because of the complementarity effects, the reduction in the use of energy in production decreases the amount of capital (k) by 1% and the amount of labour (l) by 0.5% approximately. The decrease in the productive inputs is translated into an output (Y) decrease of 2% which would imply a negative association between output (Y) and energy prices (P). Finally, consumption (c) exhibits a similar response to the output (Y).

4.2 TRANSMISSION MECHANISMS OF TECHNOLOGY SHOCKS

Dedola and Neri (2006) argue that in the standard DSGE model, technology shocks play an important role in accounting for output fluctuations. Our results

[6] The persistent of capital is not reported in the table as we mainly focus on consumption, investment and output in this table.

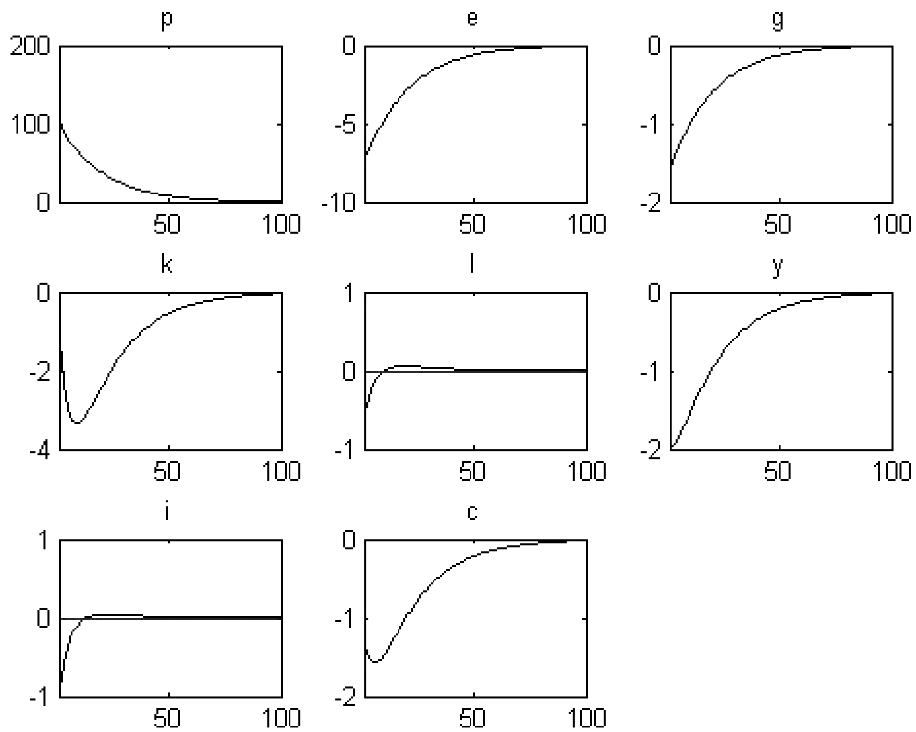

Figure 1. Impulse responses to an energy price shock.

reveal that the technology shock has stronger impact on the variables than the energy price shocks.

An increase in technology (*A*) makes capital more productive in the future. Since future technology is expected to be higher, the social planner responds optimally by immediately building up the capital stock (*k*) by 40%. As a result of a positive technology shock, investment (*i*) rises by 25% and output (*Y*) by 50%. The IRF of consumption (*c, e*) displays a hump shape as is already documented in the literature. Investment (*i*) reverts back to original pre-shock levels just after a few periods compared to other endogenous variables.

It is worth noting that the behaviours of IRF for the endogenous variables are opposite in directions to their response to an exogenous technology and energy price shock as the later shock acts as a negative technology shock. Finn (2000) also finds that an energy price shock can be considered as an adverse technology shock, since it causes capital (which embodies the technology) to produce at below capacity levels.

5. CONCLUSIONS

McCallum (1989) suggests that DSGE theory should explicitly model exogenous energy price changes. We made an attempt to implement this suggestion in the simplest possible way where energy is included both in the utility and production functions which constitute a novelty with respect to previous literature. Energy price shock is explicitly introduced in our model in addition to the technology shocks. In addition we contribute to the existing literature by modelling energy price shocks in a DSGE framework for a developing country, Bangladesh.

The main conclusion from our paper is that energy price shocks are not a major factor for macroeconomic fluctuation in the Bangladesh economy and therefore, output fluctuations in Bangladesh are mainly driven by technology shock. This might be the case of the substitution possibility of energy with labour and capital in the production process as described by Dhawan and Jeske (2007). Besides, different measures of the underground economy of Bangladesh has pointed out that the informal economy had the size of 35% of the total official GDP, which is a large value and sufficient enough to distort any macroeconomic outcomes (Schneider, 2004).

Additionally, variance decomposition analysis shows that energy price shock contributes a very small percentage (3.29%) to variations in overall output, similar to results obtained in Tan (2012), Dhawan and Jeske (2007) and Kim and Loungani (1992). It is also not surprising that a choice of functional forms and parameterisation may affect model dynamics and also change the model’s amplification and propagation mechanism (Kormilitsina, 2011). In fact, our results offer some support to the views of macroeconomists who downplay the impact of energy price shocks on the business cycle fluctuations (Dhawan and Jeske (2007). It is also worth noting

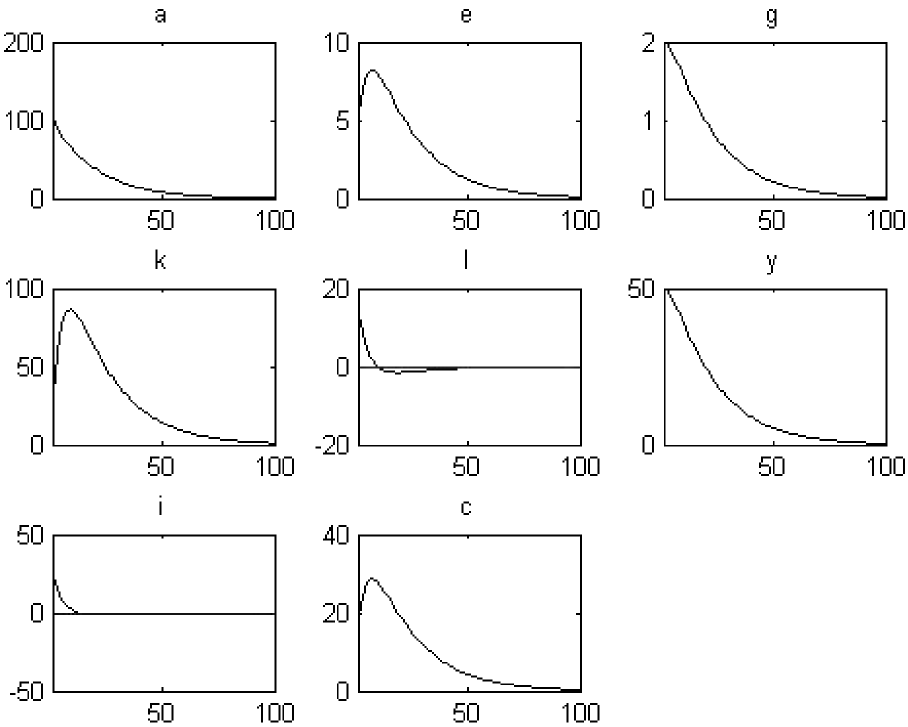

Figure 2. Impulse responses to a technology shock.

that when we scrutinise the IRF generated results in response to the exogenous energy price shocks, we may speculate an inverse relationship between different economic variables (like energy usage, productive inputs, consumption, output, etc.) and energy prices in Bangladesh economy. However, these relations are completely outweighed by the stronger positive impact of the exogenous technology shocks on the variables.

Our model could be generalised by introducing different types of households, firms, energy generating firms and a government sector to carefully analyse policy in developing countries. In fact, Jamasb (2006) argue that in most developing countries, electricity reform requires extensive restructuring of prices and subsidy arrangements. Therefore, our benchmark model could be extended by considering a detailed disaggregated electricity sector for a mixed economy where the government controls energy prices charged to households and firms, and enables the government to absorb the shocks. Consequences of energy price liberalisation can also be analysed.

REFERENCES

Ahmad, W., Haider, A., and Iqbal, J., 2012, "Estimation of Discount Factor (beta) and Coefficient of Relative Risk Aversion (gamma) in Selected Countries", Munich Personnel RePEc Archive, MPRA Paper No. 39736.

Bangladesh Bureau of Statistics, BBS, 2015.

Bangladesh Household Income and Expenditure Survey, 2015.

Bartleet, M., and Gounder, R., 2010, "Energy Consumption and Economic Growth in New Zealand: Results of Trivariate and Multivariate Models", *Energy Policy* 38, pp. 3508–3517.

Choudhary, M.A., and Pasha, F., 2013, "The RBC View of Pakistan: A Declaration of Stylized Facts and Essential Models", Discussion Paper in Economics 04/13, University of Surrey.

Cooley, T.F., and Prescott, E.C., 1995, "Economic Growth and Business Cycle", In Frontier of Business Cycle Research, Princeton, NJ.

Dedola, L., and Neri, S., 2006, "What Does A Technology Shock Do? A VAR Analysis with Model-Based Sign Restrictions", Working Paper Series No. 705, European Central Bank.

De Miguel, C., Manzano, B., and Martín-Moreno, J.M., 2003, "Oil Price Shocks and Aggregate Fluctuations", *The Energy Journal* 24 (2), pp. 47–61.

De Miguel, C., Manzano, B., and Martín-Moreno, J.M., 2005, "Oil Shocks and the Business Cycle in Europe", EEE 215.

Dhawan, R., and Jeske, K., 2007, "Energy Price Shocks and the Macroeconomy: The Role of Consumer Durables", Mimeo, Federal Reserve Bank of Atlanta.

Finn, M.G., 2000, "Perfect Competition and the Effects of Energy Price Increases on Economic Activity," *Journal of Money, Credit and Banking* 32, pp. 400–416.

Ghez, G., and Becker G., 1975, "The Allocation of Time and Goods over the Life Cycle", New York: Columbia University Press.

Griffoli, T.M., 2011, "Dynare User Guide: An Introduction to the Solution and Estimation of DSGE Models", Dynare v4 User Guide, Public Beta Version.

Hall, R.E., 1988, "The Relation between Price and Marginal Cost in U.S.Industry", *Journal of Political Economy* 96, pp. 921–947.

International Monetary Fund, IMF, 2001, "Tax Policy for Emerging Markets: Developing Countries", prepared by Vito Tanzi and Howell, H.Zee, IMF Report.

Jamasb, T., 2006, "Between the State and Market: Electricity Sector Reform in Developing Countries", *Utilities Policy* 14, pp. 14–30.

Kim, I.M., and Loungani, P., 1992, "The Role of Energy in Real Business Cycles", *Journal of Monetary Economics* 29, pp. 173–189.

King, R., Plosser, C.I., and Rebelo, S., 1988a, "Production, Growth and Business Cycles, I: The Basic Neoclassical Model", *Journal of Monetary Economics* 21, pp. 195–232.

King, R., Plosser, C.I., and Rebelo, S., 1988b, "Production, Growth and Business Cycles, II: New Directions", *Journal of Monetary Economics* 21, pp. 309–342.

Kormilitsina, A., 2011, "Oil Price Shocks and the Optimality of Monetary Policy", *Review of Economic Dynamics* 14 (1), pp. 199–223.

Liu, S., 2008, "Capital Share and Growth in Less Developed Countries", China Centre for Economic Research.

Lucas, R.E., 1990, "Supply Side Economics: An Analytical Review", *Oxford Economic Papers* 42, pp. 293–316.

McCallum, B.T., 1989, "Real Business Cycles", in: Robert Barro (ed), Modern Business Cycle Theory, Cambridge, Harvard University Press, pp. 16–50.

Prescott, E.C., 1986, "Theory Ahead of Business Cycle Measurement", *Carnegie-Rochester Conference Series on Public Policy* 24, pp. 11–44.

Rahman, S., and Rahman, A.K.M., 2002, "The Estimation of Capital Stock for the Bangladesh Economy", Focus study number 1, Micro Impacts of Macroeconomic and Adjustment Policies in Bangladesh, Bangladesh institute of Development Studies (BIDS).

Rahman, J., and Yusuf, A., 2010, "Economic Growth in Bangladesh: Experience and Policy Priorities", *Journal of Bangladesh Studies* 12 (1).

Rotemberg, J.J., and Woodford, M., 1996, "Imperfect Competition and the Effects of Energy Price Increases on Economic Activity", *Journal of Money, Credit and Banking* 28, pp. 549–577.

Schneider, F., 2004, "The Size of the Shadow Economies of 145 Countries All over the World: First Results over the Period 1999 to 2003", IZA Discussion Papers, Institute for the Study of Labour (IZA).

Tan, B.H., 2012, "RBC Model with Endogenous Energy Production", School of Economics, Singapore Management University.

Tobin, J., 1980, "Stabilization Policy Ten Years After", *Brookings Papers on Economic Activity* 1, pp. 19–71.

Yisheng, B., 2006, "Fixed Capital Stock Depreciation in Developing Countries: Some Evidence from Firm Level Data", *Journal of Development Studies* 5, pp. 881–901.

Validity of Fama and French Model on RTS Index*

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Abstract. The tools for estimating expected returns have advanced from mean-variance relationship to CAPM, a one-factor model that set the background for a more developed multifactor Fama-French model. Different developed and emerging markets were considered while testing the CAPM and the three-factor model. However, Russian capital market was lacking the Fama-French model test. This is a market with unique conditions of the transitional economy. The testing of the validity of the model on RTS was chosen as an objective for this research. With the dataset of 50 blue-chip Russian companies the results revealed that Fama-French outperforms CAPM on RTS index. Despite that, there are several limitations to the model due to the market inefficiency in Russia. This fact leaves arbitrage opportunities for investors.

Аннотация. Финансовые инструменты, позволяющие определить ожидаемые доходы, развились от простой взаимосвязи риска и доходности до CAPM и далее до трехфакторной модели. В процессе проверки моделей CAPM и Fama-French были изучены различные развитые и развивающие рынки, кроме российского рынка. Данный рынок находится в переходном состоянии, и тестирование модели Fama-French на индексе РТС было выбрано для исследования. По данным топ-50 компаний, в результате исследования было показано превосходство трехфакторной модели над CAPM на индексе РТС. Несмотря на это, существует несколько ограничений в модели из-за неэффективности российского рынка. Данный факт позволяет инвесторам использовать арбитражные возможности.

Key words: Fama-French model, RTSI, CAPM, expected-return, stock portfolio.

1. INTRODUCTION

In the world of finance, the estimation of expected returns and portfolio performance evaluation has always been a central issue for the academics and practitioners. The first major appearance of such technique was mean-variance relationship of the returns (Markowitz, 1952), followed by CAPM and the latest — widely recognized — Fama and French model. Fama and French three-factor model, which was initially set out in the fundamental Fama and French (1992) work, was a breakthrough in the financial world. It employed additional factors for size and book-to-market ratio. The tests on the developed markets followed with the majority of the studies done in the developed and emerging markets, which employed the success of the three-factor model. Still, there are some markets, which were not explored.

Russian stock market is one of the cases. It has the features of the emerging market; furthermore, there are signs of its transitional nature.

In 1992–1997, after USSR breakup, the market economy developed at unprecedented pace. The economic structure skewed towards the service sector, providing in official figures 41% for the industry and 51.5% for the service in 1995, whereas two years ago the figures favoured industry sector. In addition, the newly diversified economy was accompanied with the falling trend of GDP and inflation peaked at 2300% annually. Despite that, Russian market was appealing to the foreign investors because of its capacity and opportunities (Kvint, 1998).

Financial markets appeared in such conditions. Since 1992, MICEX (Moscow Interbank Currency Exchange) and RTS (Russian Trade System) have been the major national stock exchanges with $ 50 million traded every day back in 1998 (Kvint, 1998).

The stock markets appeared to be somewhat successful. So far, there were issues that constrained the foreign investors. The most problematic areas could be outlined as unavailability of the correct audited financial results that conform to international standards; refusal to allow shareholders to appear on board

* Оценка адекватности модели Фама и Френч на индексе РТС.

of the directors, despite law guarantees. Moreover, there were persistent non-disclosure of the trading deals, long periods of confirming the trades and, finally, restrictions of some stocks to be purchased only by Russians (*New York Times*, 1997).

Over the years, there was a significant improvement in the stock market governance and the foreign rating agencies' attitude towards Russian investment climate, according to Sollogoub (2003). For four years of high oil and gas, prices improved the Russian balance, but the economy was bound to the fossil fuel prices. In spite of the diminishing diversity of the economy, Moody's upgraded the country's rating by two notches to Baa[3] in 2003. By taking that unconventional step, the agency put national stock market in rather appealing condition, as well as the whole economy to continue improving. Still, there might be issues with the institutional aspects, as it appeared to be questionable in terms of governance (Sollogoub, 2003).

The global financial crisis revealed the problems of Russian economy. During the turmoil of 2008, MICEX and RTS plunged almost 54 percent along with oil price. Despite that, the government managed to respond quickly to prevent severe losses and to control unemployment with help of reserves. Banking system was also saved from collapsing. The Russian economy managed to sustain the crisis reasonably well (Guriev & Tsyvinski, 2010).

In 2014, Russia faced another challenge of the falling prices for the fossil fuels. Focusing on the oil and gas production, the drop in prices affected stock market as well as entire economy to shrink. The result of that was downgrading of the credit rating to near 'junk' level — Ba1 (Moody's, 2015). That might be the problem of the poor economy diversification.

So far, Russian emerging market has the process development of what have already been present on developed markets for some time, for example stock exchanges and companies' stocks. For some, the privatization appears to be the main challenge. Overall, there is highly probable form of market inefficiency.

2. LITERATURE REVIEW

The asset-pricing models were under constant development since the second half of the twentieth century. Finance practitioners and academics have been seeking better tools to predict the market as well as calculate the future cost of capital and measure portfolio performance.

The literature discussion starts with the Capital Asset Pricing Model (CAPM), which was developed by Sharpe (1964), Lintner (1965) and, consequently, Mossin (1966). This was a major breakthrough in determining the expected returns through risk-return relationship since the portfolio model was introduced by Markowitz (1952).

Markowitz's model operates under the assumption that investors want to minimize the variance of portfolio return and maximize the expected return, thus the model is known as 'mean-variance' model.

CAPM employs the central mean-variance relationship developed by Markowitz (1952). Sharpe (1964) and Lintner (1965) add two key assumptions to the mean-variance model: complete agreement on the asset returns distribution, the borrowing and lending is possible at risk-free rate. In addition, the idea market equilibrium was introduced, that there are common interest rate for investors and their expectations of the further market movements are the same (Sharpe, 1964).

CAPM employs transformation of algebraic statement of the 'mean-variance' model into the testable prediction of the relation between risk and expected return on markets in equilibrium. The formula for CAPM (1) can be incorporated in the following form by simple derivation (Jensen, 1972) from the original Sharpe-Lintner research.

$$R_m - R_f = \alpha_i + \beta\left[E\left(R_M\right) - R_f\right] + e_{it} \tag{1}$$

Despite being viable in theory, CAPM failed to provide an empirical evidence that proves the success of the model. Through several tests on explaining excessive returns Black, Jensen and Scholes (1972), Fama and Macbeth (1973), and Fama and French (1992) revealed that the model did not succeed in the explanation of the excessive returns of the securities on market, as the security market line appeared to be too flat.

Fama and French (1992) argue that CAPM might be based upon rather unrealistic assumptions, as mean and variance over the one period. Researchers suggest that market beta misses significant dimensions of risk assigned to the labour income and future investment expectations. Through cross-sectional regression approach, the findings by Fama and French suggest that the use of size and book-to-market equity helps 'to absorb the roles of leverage and E/P in average stock returns' (Fama and French, 1992, p. 428). The book-to-market equity ratio has stronger explanatory power than size, but the book-to-market ratio cannot replace size in explaining average returns.

The same conclusions appear in following papers by Fama and French (1993, 1996) with use of time-series approach. The formula used in papers as introduced by Fama and French (2):

$$\begin{aligned} R_{it} - R_{ft} = \alpha_i + \beta_{iM}\left[E\left(R_{Mi}\right) - R_{ft}\right] + \\ + \beta_{is}E\left(SMB_t\right) + \beta_{ih}E\left(HML_t\right) + e_{it} \end{aligned} \tag{2}$$

This model improved the CAPM by providing two additional factors that seem to explain the returns. SMB is the excess return on a portfolio of small stocks over a portfolio of large stocks, while HML is the excess return on a portfolio of high book-to-market stocks over a portfolio of low book-to-market stocks. Betas are the slopes in the multiple regression. If asset pricing is rational, size and BE/ME must proxy the risk (Fama & French, 1992).

Fama and French (1993) found that despite the size and book-to-market ratio are not the state variables, higher average returns on small stocks and high book-to-market stocks reflect unknown state variables that are able to price the undiversifiable risk in returns, left by CAPM model. Furthermore, according to Fama and French (1995) paper, the findings show that weak firms with prevailing low earnings tend to have high BE/ME and positive slope on HML and negative slope in case of strong firms with persistently high earnings. HML appears to capture the variation of the risk factor related to earnings performance. Coupled with SMB, there are two main conclusions that stocks with low long-term returns tend to have positive SMB and HML slopes and higher average returns. In contrast, the stocks with high long-term returns tend to have negative slopes on HML and low future returns.

The introduced model employed rather forceful techniques, which were a subject to critique in several studies. A paper by Kothary, Shanken and Sloan (1995) states that the substantial part of premium results from survivor bias. Data snooping appears to be the other issue, addressed by Black (1993) and MacKinlay (1995). Apart from that, the distress premium was claimed to be irrational as the results of investor over-reaction that lead to under-pricing of distressed stocks and overpricing of growth stocks (Lakonishok, Shleifer, & Vishny, 1994). Fama and French (1996) discussed the outlined problems and concluded that suggested improvements follow the initial results. However, there is a drawback that three-factor model could not explain the momentum effect that leaves persistence of short-term returns unexplained.

In response, a research by Carhart (1997) claimed, that three-factor model might be improved, by adding the momentum coefficient. The study included tests of the three models: CAPM, three-factor model and the new four-factor model, introduced by Carhart (3).

$$R_{it} - R_{ft} = \alpha_{iT} + \beta_{iT} RMRF_t + s_{iT}\left(SMB_t\right) + \\ + h_{iT}\left(HML_t\right) + p_{it} PR1YR_t + e_{it} \quad (3)$$

This model accommodates primarily two previous studies by Fama and French (1993) and Jagadeesh and Titman (1993). The latter study introduced the momentum factor (interpreted by Carhart as PR1YR), which captures the one-year anomalies. Through the tests, it was uncovered, that the four-factor model significantly improves CAPM as well as three-factor model. The four-factor model reduces the average pricing errors. It might be compared by actual figures: 0.35% for CAPM, 0.31% for Fama-French model and 0.14% for the Carhart model (Carhart, 1997).

In response to the Carhart research and the inability of the three-factor model to explain persistent short-term returns, Fama and French (2004) accept that lack of momentum effect as the main drawback. They, however, suggested that the sensible application of Carhart model appears to be achieving a goal of uncovering information and manager-specific effect free of known pattern in average returns. Moreover, Fama and French argue, that due to the short life of momentum effect it is likely to be irrelevant for estimates of the cost of equity capital.

Thus, Fama and French (2015) did not include the momentum in their five-factor model, as it is likely to affect the diversification of some of the portfolios used to construct the factors. This model employs another two additional factors based on the evidence of Novy-Marx (2013) and Titman, Wei and Xie (2004). These factors are profitability (RMW) and investment (CMA) and they follow the motivation that three factors of the original Fama-French model might miss the variation of the profitability and investment factors. The formula for the introduced model is following (4):

$$R_{it} - R_{ft} = \alpha_i + \beta_{iM}\left[R_{Mt} - R_{Ft}\right] + s_i SMB_t + \\ h_i HML_t + r_t RMW_t + c_i CMA_t + e_{it} \quad (4)$$

As a result the Fama and French paper reports that the explanatory ability of the model somewhat improved, capturing from 71% to 94% of cross-section variance in expected returns. However, there could be capturing of the low-average returns on small stocks that mimic the high-volume investing companies despite the low profitability. In addition to this limitation, the HML factor might become redundant as its variations are captured by the two added factors (Fama & French, 2015). Still, the paper is new, and the tests are to be conducted.

To sum up, there is still no perfect solution in explaining the return on the stock markets. The development of the theories from mean-variance to five-factor model over last decades creates better explanatory results and diminishes the limitations of the predecessors. However, Fama and French (1996, 2004) claim, that their creation is just a model and it cannot be an ultimate tool for explaining all stocks and portfolios.

FAMA-FRENCH MODEL TESTS

On the developed markets, significant amount of tests were conducted, which provided different results on explanatory ability of the Fama-French model. Despite that, it could be argued, that three-factor model is more likely to be successful.

Initially, Fama and French were the first to carry out the tests of the model in 1992. The research aimed to test the validity of their model on the Northern American stock markets (NYSE, AMEX and NASDAQ) throughout the period of 1962–1990. Only non-financial firms were included in analysis in order to provide the consistent results. The result of their study unveiled that the three-factor model proved to capture the variations associated with size and book-to-market equity (Fama & French, 1992). Fama and French claimed that there might be a chance of the practical application of the developed model as it showed the systematic patterns of low BE/ME firms to be relatively better earners comparing to high BE/ME firms.

The investigation on the same markets was revisited by Lam (2005). This study compares CAPM to Fama-French model in ability of describing the market anomalies. The comparison is conducted on 25 portfolios formed on size and book-to-market ratio and 30 portfolios, shaped by industries. These portfolios were created on NYSE, AMEX and NASDAQ stocks throughout time periods: 1926–2004 and 1963–2004. The ordinary least square linear regressions econometric technique was used. The research produced questionable results, as three-factor model could not explain market anomalies on the 1963–2004 time period for 30 industries. This paper also reveals that Fama-French three-factor model might be portfolio specific, test specific as well as period specific (Lam, 2005).

The study by Hussain, Toms, & Diacon (2002) provides an accurate test on London Stock Exchange, similar to original Fama-French (1996) paper. Despite the differences in database, slightly different grouping of variable, the research provides strong evidence in favor of the Fama French three-factor model over CAPM. The R-squared is 0.59 and 0.83 for CAPM and Fama-French model respectively on average of the 25 regression (Hussain *et al.*, 2002).

Another research is conducted by Faff (2004) on a rather remote developed market. This paper tests the Fama-French model on the Australian stock market. The researcher argues that although this market is small, it is developed enough to provide adequate results for three-factor model test over the dataset from 1996 to 1999 on approximately 320 Australian companies. This paper provides quite favourable results for the validity of Fama-French model. However, the validity deteriorates when the estimated risk premia is considered, leaving the negative size premiums uncovered. In addition, there is a concern in the study about the data snooping and reliance on the index data from Frank Russell Company (Faff, 2004).

In summary, the evidence from developed markets favours the three-factor model and follows the Fama and French (1992, 1993, and 1996) papers.

FAMA-FRENCH FEATURES OF EMERGING MARKETS

Emerging markets present an opportunity to conduct out-of-sample test of the model. According to Fama and French (1998) on the emerging markets the significance of BE/ME and returns relationship persist. That confirms the pervasive nature of the value premium and follows the evidence from the developed markets. The size effect could be observed in emerging market returns, as small stocks possess higher average returns than the big stocks in eleven out of sixteen of the markets analysed. However, the research shown high volatility on the markets and short sample period, which diminishes the ability of the study to produce accurate results.

The empirical tests conducted by other researchers on the emerging markets produced controversial results. The study by Eraslan (2013) revealed limited explanatory power to explain excessive returns of stocks listed on ISE (Istanbul Stock Exchange) from 2003 to 2010. The similar result is produced by another research (Soumare, Amenounve, Meite, & N'Sougan, 2013), which revealed limitations of three-factor model on explaining the BRVM market returns on African market throughout the 2001–2008 period. In contrast, the results from Karachi stock exchange (Rafi, Kazmi, & Haslim, 2014), stock exchange on Mauritius (Bundoo, 2008) uncovered the validity of the three-factor model on these markets.

The studies were carried out mostly using the Fama-French (1993) sorting technique, with deviations in order to meet the country specifics. For instance, in the research employed on the BRVM (Soumare, Amenounve, Meite, & N'Sougan, 2013) median market capitalization was used as the breakpoint for the size, and the 30th and 70th percentile as benchmark to distinguish book-to-market values into three categories of the companies. It appeared slightly different in the research on Mauritius stock market (Bundoo, 2008), which distinguished only two classes of book-to-market ratio on median value. This approach was more suitable due to the smaller sample size.

The research by Eraslan (2013) on Istanbul stock market was made on the similar methodology to Fama and French (1996). Firms were allocated in three groups by the low 30 percent, medium 40 and high 30 percent on every variable. Then, nine portfolios were constructed from 274 stocks and were sorted into six portfolios. The results were that medium size portfolios tend to outperform the portfolios of smaller sizes, although, it seems to be unsatisfactory to assess big size portfolios. Moreover, it could be noted, that the conducted study carried less power to assess the validity of the discussed model in comparison to the others, being done on the ISE. That may be explained by the different time periods, number of stocks in the portfolio, and by the inclusion of global financial crisis in the analysed period (Eraslan, 2013).

Almost the same result appears in another research that considers African stocks on BRVM over the period from 2001 to 2008. The correlation between emerging African market and the developed markets is low. Thus, the research aims to find an explanation of the stock returns in light of market imperfections, such as poor governance structure, inadequate investor protection etc. This study presents that Fama-French model explains returns for 10 out of 28 stocks, or 35.71%; so there appears to be limitations for the validity of the discussed model (Soumare *et al.*, 2013).

The limitations of three-factor model were unveiled in the study on Karachi stock exchange market. The research showed limited results in favour for the Fama-French model (Rafi, Kazmi, & Haslim, 2014) on the KSE-100 in the period of 2011–2013 on 100 companies, sorted by the same technique as implemented by Fama and French (1992). The results of the research shows that the three portfolios valid for the market risk premium, four for the size premium and three portfolios valid for all factors. However, it could be argued, that the three-factor model might not be able to successfully describe the excessive return in the KSE-100 index, as four out of six portfolios possess insignificant results to their intercepts.

In contrast to previously discussed papers, the study by Bundoo (2008) investigates the validity of the Fama and French on SEM (Stock Exchange of Mauritius). The research takes into account 40 stocks, from the period from 1998 until 2004. Number of the companies varied from 6 (1998) to 40 (2004). The author implemented the augmented three-factor model, which considers time-variance factor. The result of this paper produces the evidence of the validity of Fama-French model on the SEM. That also brings in the empirical evidence on emerging markets.

In Eastern European emerging markets, the research conducted by Foye, Mramor and Pahor (2013) addresses the issue of probable data mining rather than appropriate proxy for risk for the three-factor model since its origin in 1992. The discussion of this issue is influential in European nations with emerging market that joined the EU in 2004 (Foye *et al.*, 2013). This research tests the validity of the model on stock market in several countries: Poland, Hungary, Czech Republic, Slovenia, Slovakia and Baltic countries over the period 2005–2012. Through the work, the researchers found three-factor model to follow the results of Fama and French (1993) paper for book-to-market ratio factor, whether for size the slope coefficients appeared to be negative and of low explanatory ability.

These results corroborate the earlier study by Claessens, Dasgupta and Glen (1995) which provides the evidence that size effect could not be fully reliable in explaining the market returns, as only eight out of nineteen emerging markets produced the highest rate of stock portfolio returns. In addition, it revealed the highest standard deviation, which the market returns seem not to be related to the size and inaccurate predictions.

Foye *et al.* (2013) provided extension for the Fama and French (1998) and Claessens *et al.* (1995) findings and instead of size factor suggested using the LMS coefficient based on NI/CFO, or net income to cash flow from the operations. This factor does not support the investors with relevant information about the company's performance, however, it still might be useful for indicating the 'earnings quality' (Foye *et al.,* 2013, p. 15) and accounting manipulation. The investors might evaluate the differences between net income and cash flow from operations as being associated with accounting manipulation. Thus, the proposed coefficient might represent the risk factor (Foye *et al.,* 2013).

By using the new factor, the study produced better results, comparing to the three-factor model. The NI/CFO factor returns appear to provide significantly higher R^2 values than the model employed market equity factor. The figures of adjusted R^2 are on favour for the NI/CFO (0.13) rather than for ME (0.03). In addition, with the new factor employed, the direction of the regression slopes does not change considerably, whether the slope coefficients for the ME are negative for the low-ME (from –0.51 to –0.81) and positive for the high-ME (0.72–1.15) (Foye *et al.*, 2013). It appears that proposed factor presented better explanatory power to the eastern European stock markets. However, this model is new and the initial research was conducted on the number of countries, whereas this article focuses only on one country. Thus, the developed model by Foye *et al.* (2013) might to be rather unsuitable for the current research paper.

Generally, the tests of three-factor model on emerging markets follow the results from developed markets, providing the same BE/ME effects and rather limited size effect. The limitations might be related to the short time periods or sorting method.

RUSSIAN STOCK MARKET

Russian market is one of the largest among the emerging market countries because of large territory, high capacity and high market capitalization. However, it is still in transition to the conventional market economy. This process involves specific challenges and Russian stock market might share the discussed problems of emerging markets. Yet, there could be unique risk factors that influence the market performance.

A paper by Goriaev and Zabotkin (2006) investigates the risks assigned to the Russian stock market in first decade after it was created. Started from a scratch in 1994, the Russian stock market had a total capitalization over $ 600 bn or 80% of GDP at the end of 2005. As authors claim, this result was achieved after resolving two important challenges.

The first challenge is macroeconomic stability that was influenced by the recovery of oil price and prudent fiscal policy. The second challenge is a political stability assumed by Putin's legitimacy and popularity. His meeting with business leaders in 2000 resulted in settlement of the incentives for the corporate governance. This appeared to be a turning point after the privatization process, as these incentives improved security of major companies' assets in Russia. Thus, the business owners were given an interest in both maximizing and protecting their wealth and improved reputation.

As Goriaev and Zabotkin (2006) observe RTSI's performance over the first decade, when the discussed issues were addressed, the overall progress of transition persisted. The evidence of this was the creation of value in companies in the commodity-exporting sectors, dominated in the economy, and the emergence of the new business that was consumer-oriented and could not exist in USSR (Goriaev & Zabotkin, 2006). Goriaev and Zabotkin (2006) claim that short-run movements in Russian stocks might be linked to the fluctuations in domestic and international markets, including commodity markets (crude oil in particular), global equity markets and foreign exchange.

It was a period of development of stable links with macroeconomic variables during 1995–2004 and overall maturing of the market; however, the study by Anatolyev (2005) argues that Russian stock market became more sensitive to global factors, such as the U.S. stock market performance and interest rates. The study by Peresetsky (2014) provides evidence that Japan market is more significant to Russia at least over the period 2000–2010. This is because the closures of the Russian and Japanese markets are close to each other in time, whether the US market is too far. Peresetsky (2014) claims that NIKKEI index contains more relevant information, which might possess predictive power for the Russian equity market.

Other driver — oil price — plays important role in Russian economy, and it might influence the stock markets as oil price expectations in long-term are gradually reassessed, whereas interim oil price volatility has a secondary importance on the emerging markets. A paper by Rozhkov (2005) states that about 60 percent of RTS index's performance is determined by oil prices, in other words, 60 percent is allocated to oil producers. It can be argued that oil price might be evaluated as the most important market driver and it also carries a large risk (Rozhkov, 2005). However, another research by Peresetsky (2014) of market drivers on the period of 2000–2010 unveiled the vanishing significance of oil prices for the stock market after 2006. This conclusion might be considered rather controversial in light of the latest events. The drastic drop in oil prices depreciated rouble and hence created inflation in economy (*The Economist*, 2014).Then the capital market shrank to total capitalization of $ 531 bn, which is less than a market capitalization of Apple — $ 669 bn (Tadeo, 2014).

The final factor is foreign exchange rate, which contributed, according to Goriaev and Zabotkin (2006), to the growth of the RTSI. The estimated coefficients for RUB/USD were significant by 21% and for USD/EUR by 34%. The impact of foreign currencies' rates appeared to be the most evident from 2000 to 2005. As a result, the exporting companies seemed to get the largest benefit (Goriaev & Zabotkin, 2006). Similar results were obtained by Saleem and Vaihekovski (2008), who found currency risk to be a separate risk factor on Russian stock market over the period 1995–2006. In contrast, the study by Kinnunen (2012) unveiled little explanatory power for the expected return on the Russian market through application of conditional multifactor and autoregression model over period 1999–2012.

All in all, the Russian stock market has a predictable volatility in different sectors of economy, claims the study by Saleem (2014) on period of 2004–2013. Using FIGARCH model, the paper establishes stock market long memory in all sectors of the Russian capital market, which moves to the implication that the modern Russian equity market is weak form efficient. This results are consistent with the earlier work by Anatolyev (2005), which also found Russian market instability to be not confined to the financial crisis. Saleem (2014) concludes on the need of regulatory and economic reforms within national financial system. So far, there are arbitrage opportunities for international investors.

OBJECTIVE

This article aims to test the Fama and French three-factor model on Russian stock index RTS. Given the challenging environment of this market, associated features, and lack of literature investigating the Fama-French model on Russian equity market, the research on it would enrich the overall evidence from the promising emerging markets. Apart from that, that would help finding the ability of the three-factor model to explain the Russian market.

RTS is fundamental market index calculated on prices of the 50 most liquid Russian stocks of the largest and dynamically developing Russian issuers presented on the Moscow Exchange. RTS Index was launched on September 1, 1995 at base value 100. It is calculated in real time and denominated by Moscow Exchange in US dollars, which is an adjustment of MICEX index values by the current exchange rate. The market capitalization was $ 116 bn by the end of 2014 (Moscow Exchange, 2015).

Motivation of the research to investigate this particular index is the US dollar denomination that makes this index interesting for the foreign investors, as it would provide rather clear picture of the current situation in Russian economy.

3. METHODOLOGY

The preferred method of the research still would follow Fama and French techniques, as it investigates the emerging type of market in a country with its own features, where three-factor model was not tested previously. This research would implement the process of portfolios construction that follows the Fama-French (1993) approach. There are three factors in the model equation that should be provided with appropriate data.

$$E(R_{it}) - R_{ft} = \alpha_{it} + \beta_{iM}\left[E(R_{Mt}) - R_{ft}\right] + \\ +\beta_{is}E(SMB_t) + \beta_{ih}E(HML_t) + \varepsilon_{it}$$

The first factor $[(R_{Mi})-R_{ft}]$ acts as proxy to the excess market portfolio return. RTS index is used as this proxy. The second factor is small minus big (SMB), which provides the difference in returns between a portfolio of small stocks and a portfolio of big stocks. The final factor is high minus low (HML), which represents the difference between high book-to-value (BE/ME) stocks and low book-to-market value (BE/ME) stocks. To avoid any confusion, the small and big are associated with the market equity (ME) which is the total shares on the market and the share price. The low and high relate to the book-to-market value that shows the relation between the book value and market value of the share. Book value is the accounting measure of 'net worth of the company as reported on its balance sheet' (Bodie, Kane, & Marcus, 2011).

The following step would be to create portfolios from the combination of the market size and book-to-market value. That would be implemented by sorting RTS stocks independently in the two size groups (low and high) and three book-to-market equity (BE/ME) groups: low, medium and high (L, M and H). The breakpoint for the size would be a group median of the dataset. The breakpoints to distinguish the BE/ME groups would be 30th percentile for low and 70th percentile for the high. The middle group would be situated in-between 30th and 70th percentiles accordingly. Finally, there would be six portfolios created on the intersection of the two market equity groups and three book-to-market value groups. These portfolios would be S/L, S/M, S/H, B/L, B/M and B/H.Each of these portfolios should have stocks that could be attributed to the both categories, e.g. high BE/ME and small size stock would be placed in the S/H portfolio.

ESTIMATION OF EXCESS RETURNS

The excess returns would be sourced by all three factors (market, SMB and HML). The excess market return is estimated by the difference between market return (with dividends) of the RTS index and the risk free rate with the following formula $[(R_{Mi})-R_{ft}]$, estimated for each month. RTS index return is calculated using formula below:

$$Market\ Return = \frac{Close\ price_t - Close\ price_{t-1}}{Close\ price_{t-1}}$$

The R_{ft} factor is the return of 10-year Russian government bond, collected from Bloomberg database (Bloomberg L.P., 2015).

After market return estimation, the following procedures of forming SMB and HML factors should be carried out. There are six portfolios, (S/L, S/M, S/H, B/L, B/M and B/H) which are filled with appropriate companies. The next step is to calculate the returns during the period of observation in each group of stocks with the following technique. Every month the return of portfolio is estimated as an all-stock average return of that period. This process is carried out every year of observation period across all constructed portfolios.

The following step would be estimation of SMB and HML factors. According to the Fama and French (1993) study, the following formulas below should be employed in calculations.

$$SMB = \frac{(S/L + S/M + S/H)}{3} - \frac{(B/L + B/M + B/H)}{3}$$

For each month SMB is the difference between the averages of returns on three small-stock portfolios (S/L, S/M, S/H) and three big-stock portfolios (B/L, B/M and B/H).

$$HML = \frac{(S/H + B/H)}{2} - \frac{(S/L + B/L)}{2}$$

As for HML, the process appears to be similar. For each month HML is the difference between average returns on two high book-to-market value portfolios and two low book-to-market portfolios.

Described techniques have some limitations, as accepted by Fama and French (2004). SMB and HML factors would rather be forcefully constructed and appear not to be naturally involving investors' interest. The study by Michou *et al.* (2007) discovers the link between portfolio construction design and the estimation outcome. Thus, the results of this article are largely influenced by the factor construction design.

Despite that, the SMB and HML factors still might be useful, as they would describe the stock factors to be explanatory with size or book-to-market value.

The following step would be a multiple regression analysis that involves the ordinary least squares approach. The dependent value is the excess return of one out of six portfolios and the independent values are market returns, SMB and HML. All coefficients that would appear next to the factors should be able to mathematically explain the excessive returns inside the Fama-French model. In order to define the statistical significance the autoregression and heteroscedasticity tests would be carried out.

HYPOTHESES

Finally, there are the hypothesis tests with presumptions that the RTSI possesses the market, size and book-to-market effects; the test is robust and three-factor model works better that the conventional CAPM. The decision on accepting or rejecting the hypothesis is based on meeting the objectives set out previously.

Earlier papers have conducted the test of the Fama-French model on markets, which differed in terms of location and types. There appears to be a gap in constructing and testing the three-factor model on the Russian stock market, which this paper aims to fill due to its features and differentiation from other emerging markets.

Therefore, the main question of this research is how efficient the Fama-French model is in explaining the stock returns on the companies-constituents of RTSI index.

By testing the validity of the model, the process of its application would be followed by the hypotheses.

The **hypothesis 1**: There is a market, size and book-to-market effects on RTSI. Null hypothesis H_0: the coefficients of the three factors (market, size and book-to-market risk factor) equals to zero (β_{iM}, β_{is}, $\beta_{ih} = 0$). Alternative hypothesis H_1: the coefficients of the three factors is different from zero (β_{iM}, β_{is}, $\beta_{ih} \neq 0$).

The **hypothesis 2**: Fama and French three-factor model is robust on RTSI. Null hypothesis H_0: the coefficients of the three factors (market, size and book-to-market risk factor) equals to zero simultaneously (β_{iM}, β_{is}, $\beta_{ih} = 0$). Alternative hypothesis H_1: there is at least one coefficient of the three factors, which is significantly different from zero (β_{iM}, β_{is}, $\beta_{ih} \neq 0$).

The **hypothesis 3**: The Fama and French three-factor model is better than traditional CAPM model in describing the expected returns of the portfolios. Null hypothesis H_0: the Adjusted R-Square of Fama-French three-factor model and CAPM are not statistically different. Alternative hypothesis H_1: the Adjusted R-Square of Fama-French three-factor model is greater than that of CAPM model.

The **hypothesis 4**: Fama-French model is efficient in explaining the excess returns on RTS index. The null hypothesis H_0: the intercepts of regression model are equal to zero or insignificantly different from zero ($\alpha_{it} = 0$). The alternative hypothesis H_1: the intercepts of regression model are different from zero ($\alpha_{it} \neq 0$).

4. RESULTS

The estimation results are corrected for autocorrelation and heteroscedasticity using Breusch-Godfrey LM test and White test respectively. The results of both tests are summarized in the following Table 1.

The results present that two portfolios could be considered to have heteroscedasticity phenomenon — S/M and B/M portfolios. B/M deviates rather insignificantly from the critical value, but still in area of null hypothesis, whether the S/M portfolio has a large difference. Thus, the S/M portfolio's model should be reconstructed, followed by the regression analysis, while B/M portfolio might be accepted to have homoscedasticity.

For the majority of observed portfolios, the LM test has shown no sign of autocorrelation. Only for S/H portfolio for CAPM, the autoregression test revealed the negative correlation of errors. According to (Tabachnick & Fidell, 2013) it makes the estimates

Table 1. LM and White's Test Results.

White's test			Breusch-Godfrey LM test	
Portfolio	Fama-French	CAPM	Fama-French	CAPM
S/L	0.9354	**4.2385**	0.5576	0.4021
S/M	**10.6971**	0.0136	1.7844	0.3699
S/H	0.1094	6.5199	2.8501	**6.6677**
B/L	2.7014	0.0787	3.2029	1.522
B/M	0.2404	2.6339	2.2133	5.0884
B/H	1.9095	1.4208	3.1549	1.9095
Critical chi-square value	**2.71**		**9.49 (5%)**	**5.99 (5%)**
			13.28 (1%)	**9.21 (1%)**

too large, and results in loss of power. Thus, the regression procedure associated with the S/H portfolio should be modified to remove the effect of autocorrelated errors.

Overall, the verification process unveiled the issues with S/L, S/M and S/H portfolios that should be resolved by appropriate methods.

Because of the corrections and consequent estimation, the regression statistics with homoscedasticity and without autoregression are summarized below (Table 2). However, for CAPM the heteroscedasticity persists even after adjustments.

SMB and HML correlation is 0.217556, which shows the effective portfolio construction.

TESTING HYPOTHESES

After establishing that the collected data is reliable, the hypotheses could be addressed.

The hypothesis 1: There is a market, size and book-to-market effects in RTSI.

A t-test statistic could help finding the answer to this hypothesis. In the regression model the independent variables were market return risk, size and book-to-market value factors; the dependent variable are 6 portfolio returns (S/L, S/M, S/H, B/L, B/M, B/H) used one by one in each iteration. The results of regression analysis are summarized in the Table 3 below.

It could be observed that all risk factors slopes are different from zero, so the null hypothesis could be rejected. As a result, there are market, size and book-to-market effect to be on RTS index.

Interestingly, the market risk factor slopes hold the level of significance at 1% across all six portfolios. Moreover, it could be observed that the figures of β_{iM} do not differ significantly from 1. That appears to be the evidence of additional factors, as Fama and French (1993) claimed. SMB and HML hold somewhat explanatory power on RTS index.

The size slope for SMB factor provides rather controversial results. Only three out of six portfolios (S/L, S/M, S/H) provide positive results with the significance level of 1%. The slope for B/L failed the significance test; the B/M portfolio has shown the significance at 10% level and β_{is} coefficient for B/H appears to be the most accurate with 1% significance. However, all three portfolios with big size stocks appear to be negative, which means lack of the size effect for them irrespectively of level of significance. Thus, there is clear evidence that there is size effect present on RTSI for portfolios with small size stocks, and there is no size effect for portfolios with big size stocks.

Finally, a book-to-market value risk (HML) factor also possesses controversy in the obtained results. Only three portfolios (S/M, S/H, B/H) have positive results with level of significance of 5%, 1% and 1% respectively. Others (S/L, B/L) possess negative figures with 1%, 5% significance levels, while the result for B/M is not significant even for 10% level. Thus, it could be argued, that the book-to-market value effect is present only in three out of six portfolios.

However, in spite of that, it can be claimed that the behaviour of stocks cannot be explicitly explained. The evidence of that is that the regression coefficients are higher for the S/H than for B/H, whereas the implication is made on opposite (B/H > S/H). This means lack of powerful explanation of book-to-value effect, which seems to work efficiently only for the high book-to-market portfolio. In addition, the impact of book-to-market value on excess returns has unsystematic behaviour in the observed portfolios.

To conclude, the market factor appears to be statistically significant at 1% level across all observed portfolios. The SMB factor has proved the presence of size effect on RTSI only for the portfolios of small

Table 2. Adjusted Regression Estimation.

Adjusted Regression Results						
Fama-French					CAPM	
Factors	α_{it}	β_{im}	β_{is}	β_{ih}	α	β_{im}
S/L	1.1303***	1.0030***	0.7128***	−0.8329***	−2.039***	3.334***
S/M	−0.1960	0.8861***	1.1999***	0.2139***	2.819**	1.144***
S/H	0.1579	0.9752***	0.9293***	0.6837***	4.554***	1.117***
B/L	0.3083*	0.9942***	−0.0143	−0.1600**	−0.174	0.995***
B/M	1.0841*	1.0186***	− 0.1915*	−0.0442	0.82	1.000***
B/H	1.2807*	1.0220***	− 0.2308***	0.3234***	2.075***	0.996***

*	Significant level of 10%
**	Significant level of 5%
***	Significant level of 1%

White's test		
Portfolio	**Fama-French**	**CAPM**
S/L	0.9354	**7.7484**
S/M	1.1699	0.01362
S/H	0.1094	**6.5199**
B/L	2.7014	0.0787
B/M	0.2404	2.6339
B/H	1.9095	1.4208
Critical chi-square value	2.71	

Breusch-Godfrey LM test		
	Fama-French	**CAPM**
S/L	0.5576	0.4021
S/M	1.7844	0.3699
S/H	2.8501	0.6079
B/L	3.2029	1.5220
B/M	2.2133	5.0884
B/H	3.1549	1.9095
Chi-square	9.49 (5%)	5.99 (5%)
	13.28 (1%)	9.21 (1%)

size stocks, while excessive returns associated with the big size portfolios appear to have no relationship with size effect of Fama-French model. As for the HML factor, the results follow the Fama and French (1993) study as three out of six portfolios (S/M, S/H, B/H) provide positive and statistically significant (5%, 1% and 1%) slopes. It is reasonable to claim, that the model works well in explaining excess returns on RTS index after the global financial crisis in 2008.

The following hypotheses would be addressed simultaneously, as they share practically similar statistical approach.

The hypothesis 2: Fama and French three-factor model is robust on RTSI.

Table 3. Corrected Fama-French Model Regression Coefficients.

Fama-French Coefficients				
Factors	α_{it}	β_{im}	β_{is}	β_{ih}
S/L	1.1303***	1.0030***	0.7128***	– 0.8329***
S/M	–0.1960	0.8861***	1.1999***	0.2139***
S/H	0.1579	0.9752***	0.9293***	0.6837***
B/L	0.3083*	0.9942***	–0.0143	– 0.1600**
B/M	1.0841*	1.0186***	– 0.1915*	–0.0442
B/H	1.2807*	1.0220***	– 0.2308***	0.3234***

The hypothesis 3: The Fama and French three-factor model is better than traditional CAPM model in describing the expected returns of the portfolios.

Hypothesis 2 implies robustness test of the three-factor model, which will be carried out by comparing the produced p-values for the observed portfolios. In order to minimize the probability of type II error, a significance level at 5% (0.05) would be chosen. As presented in Table 4 the p-values across all portfolios are lower than 0.05. Thus, the null hypothesis should be rejected, which means the robustness of the model on RTSI.

The hypothesis 3 aims to compare two models' explanatory ability on the discussed market. The p-values for both models favour their statistical significance, but there is a major distinction in terms of R^2. It appears to be higher for Fama-French model than for CAPM in every portfolio. In addition, the R^2 are substantial and do not vary greatly (86% — 94%), while CAPM is more inconsistent (43% — 90%). On average, it can be observed that Fama-French model explains 91% of excess returns compared to 75% of returns explained by CAPM. Hence, the null hypothesis might be rejected straightaway for the market in discussion.

In addition, the final **hypothesis 4** should be considered that Fama-French model is efficient in explaining the excess returns on RTS index. It is common implication that the model performs well in explanation of the excess return, if the intercept is zero or deviates from it insignificantly. That would mean that the pricing error of such model is subtle or relatively low. The intercept comparison is presented below (Table 5).

The results provide ambiguous results. Clearly, it can be observed that Fama-French on average possesses lower pricing error than CAPM, 0.6275 and 1.3425 respectively. In addition, the half of observed intercepts is higher for Fama-French than to CAPM in corresponding portfolios. Only in three instances, CAPM outperforms the Fama and French model. For the S/L portfolio the intercepts for CAPM and Fama and French are –2.39 and 1.1303; for B/L portfolio are 0.3083 and –0.1741; and for B/M portfolio are 1.0841 and 0.8201. Interestingly, the same portfolios hold the negative figure for HML factor. Thus, intercepts might explain that book-to-market risk factor is absent for these three portfolios. In addition, the higher values of intercepts for these portfolios might be explained as the result of minimization of HML slope impact on the regression model.

Table 4. Regression Statistics for CAPM and Fama-French model.

	R squared		F-Statistics		P-value	
Factors	**Fama-French**	**CAPM**	**Fama-French**	**CAPM**	**Fama-French**	**CAPM**
S/L	94%	75%	346.611	208.402	0.00000	0.00000
S/M	90%	43%	113.510	148.978	0.00000	0.00000
S/H	93%	72%	311.552	129.542	0.00000	0.00000
B/L	89%	88%	182.433	519.778	0.00000	0.00000
B/M	86%	85%	137.601	391.692	0.00000	0.00000
B/H	93%	90%	310.675	600.156	0.00000	0.00000

Table 5. Intercepts for Regression Models.

Intercepts		
	Fama-French	CAPM
S/L	1.1303	−2.039
S/M	−0.1960	2.819
S/H	0.1579	4.554
B/L	0.3083	−0.174
B/M	1.0841	0.820
B/H	1.2807	2.075

In contrast, the S/H portfolio appears to be the most successful in explaining excess return, as the intercept's difference from zero might be claimed to be relatively insignificant, comparing to the others. Furthermore, as it was discussed earlier, the S/H portfolio holds positive slopes for risk factors with the significance at level of 1%. Moreover, by analysing the intercept, it can be concluded that S/H portfolio is the most successful example of Fama and French model on RTS index.

In summary, the hypothesis 4 should be answered with the alternative hypothesis H_1 that the intercepts of Fama-French model regressions are not equal to zero.

FINDINGS

Considered hypotheses provided the results that can evaluate how efficient Fama-French model is in explaining the returns of stock on RTS index. The outcome of the regression analysis provided limited explanatory power of Fama-French model on RTS index, as the intercepts of the regressions are significantly different from zero for 3 out 6 portfolios. Apart from that, there are size effects for small-stock portfolios and BE effect for S/M, S/H and B/H portfolios. The results are controversial and there is rather unsystematic behaviour of factors on market. That is another limitation for the Fama-French model on Russian market. However, it might be accepted that three-factor model is successful on the Russian market, as it presents larger average R^2 comparing to the result of CAPM (91% to 75%), and it has the strongest explanatory power for 3 out of 6 portfolios (S/M, S/H, B/L).

The findings of this study are similar to the conclusions from several earlier papers from emerging markets. Study by Eraslan (2013) also finds the limited explanatory power of Fama-French model on ISE, as it omits the systematic behaviour of HML and SMB limit in explain big-size portfolios. Another research by Soumare *et al.* (2013) has found that the Fama-French model outperformed CAPM on BRVM market, which is similar to the current research findings. On BRVM, three-factor model explanatory power appeared to be limited as it failed to explain the variation of returns of more than 60% of stocks. The findings by Al-Mwalla & Karasneh (2011) on Amman Stock Exchange are also close to the those from RTS index. CAPM loses to Fama-French model in explanatory power. And, finally, the evidence from Pakistani stock market (Hassan & Javed, 2011) appeared to be the closest, as high BM stocks outperform low BM stocks, as well as the inconsistency is also presented in size effect.

The limitations of size factor on RTSI follow the research papers by Claessens *et al.* (1993, 1998). These studies reported that in emerging markets the market equity factor has less explanatory ability than in developed capital markets. Fama and French (1998) also found this limitation, but despite diminished role of SMB factor, the book-to-market ratio has significant relationship with returns. That is partly true for the Russian capital market, as book-to-market value factor is present in half of observed portfolios but inconsistent.

5. CONCLUSION

The Russian stock market seems to be promising but it is rather unstable. This instability appeared not to be driven only by the financial shocks. The discussed risk drivers provide evidence, that the volatility of the Russian equity market has long memory, and it has weak form efficiency, which leads to arbitrage opportunities for foreign investors.

Fama and French three-factor model was broadly tested both on developed and emerging markets. However, the Russian capital market was lacking the application of this model. As other emerging markets, the Russian market has weak-form efficiency. Moreover, this challenging condition makes it a good area to test the validity of the three-factor model.

The results from estimation appeared to be favourable for three-factor model, as the additional two factors of Fama-French model seem to improve the explanatory ability of the traditional CAPM. The Fama-French model is presented on Russian stock market with size and book-to-market effects, however, the behaviour of the stocks of different size and BM ratio is rather unsystematic. Despite that, the three-factor model is robust, as all factors are different from zero, and it performs better than the conventional Capital Asset Pricing Model by comparing average R^2 figures for the observed portfolios. Furthermore, the intercepts' figures appeared to be rather ambiguous. The regression model, that was applied to test the model, employed significant positive slopes, which is an evidence of adequate regression application. Overall, the model appears to be valid only for 3 out of 6 portfolios, which is a limited success of the model but consistent with papers on testing three-factor model on other emerging markets.

The limited validity of the model would resolve in conclusion, that size and book-to-market ratio might be proxies for risk on a particular market. Hence, there is an implication that average returns compared to historical average benchmark have limitation in evaluating managed portfolios as well as estimating expected returns. The possible explanation of this is weak-form efficiency of the Russian capital market. In this case, the persistence of the results is more likely to be suspicious (Fama & French, 1992). Certainly, the efficiency of the Fama-French model on RTS is limited. Three-factor model still might be used to evaluate portfolio performance; however, it would not absorb all risk factors efficiently. As it was stated above there is a room for arbitrage opportunity, thus investors might try beating the market.

The received results might also be subjected to the shortcomings of the research design. One of the possible limitations of this research is sorting method, which greatly influences the outcome of the study, according to Michou *et al.* (2007). The other is the date of portfolios formation, which also has an impact on the results of the study (Michou *et al.*, 2007). Moreover, the article was focused on RTS index, which is constituted by 50 'blue chip' stocks, omitting other equities present on Russian capital market. Finally, article considered the RTS index involved USD/RUB exchange rate, which might have influenced the real movements of the market, thus distorting data for the Fama and French model.

Further work on Russian stock market might involve application of the same three-factor model using different sorting technique and including more companies in the study, as well as introducing the five-factor model (Fama & French, 2014), as well as introducing other risk factors, such as NI/CFO, suggested by Foye *et al.* (2013).

REFERENCES

Al-Horani, A., Pope, P., & Stark, A. (2003). Research and Development Activity and Expected Returns in the United Kingdom. *European Finance Review* (7), 27–46.

Al-Mwalla, M., & Karasneh, M. (2011). Fama & French Three Factor Model: Evidence from Emerging Market. *European Journal of Economics, Finance and Administrative Sciences* (41), 132–140.

Anatolyev, S. (2005). A Ten-Year Retrospection of the Behavior of Russian Stock Returns. *BOFIT Discussion Paper* (9).

Black, F. (1993). Beta and Return. *Journal of Portfolio Management* (20), 8–18.

Black, F., Jensen, M.C., & Scholes, M. (1972). The Capital Asset Pricing Model: Some Empirical Tests. *Studies in the Theory of Capital Market*, 79–121.

Bloomberg L.P. (2015). 10-year Russian governement bonds. Retrieved March 3, 2015.

Bodie, Z., Kane, A., & Marcus, J.A. (2011). *Investments and Portfolio Management* (9th ed.). New York: McGraw-Hill.

Brooks, C. (2002). *Introductory Econometrics for Finance.* Cambridge: Cambridge Unversity Press.

Bryman, A., & Bell, E. (2011). *Business research methods* (3rd ed.). Oxford: Oxford University Press.

Bundoo, S.K. (2008). An augmented Fama and French three-factor model: new evidence from an emerging stock market. *Applied Economics Letters, 15*, 1213–1218.

Carhart, M.M. (1997). On Persistence in Mutual Fund Performance. *Journal of Finance, 52* (1), 57–82.

Claessens, S., Dasgupta, S., & Glen, J. (1993). Return Behavior in Emerging Stock Markets. *World Bank Discussion Papers* (228), 323–330.

Claessens, S., Dasgupta, S., & Glen, J. (1995). Return Behavior in Emerging Stock Markets. *The World Bank Economic Review, 9* (1), 131–151.

Claessens, S., Dasgupta, S., & Glen, J. (1998). The Cross-Section of Stock Returns: Evidence from Emerging Markets. *Emerging Markets Quarterly* (2), 4–13.

Dimson, E., Nagel, S., & Quigley, G. (2003). Capturing the Value Premium in the United Kingdom. *Financial Analysts Journal* (59), 35–45.

Eraslan, V. (2013). Fama and French Three-Factor Model: Evidence from Istanbul Stock Exchange. *Business and Exonomics Research Journal, 4* (2), 11–22.

Faff, R. (2004). A simple test of the Fama and French model using daily data: Australian evidence. *Applied Financial Economics*, 83–92.

Fama, E.F., & French, K.R. (1992). The Cross-Section of Expected Stock Returns. *The Journal of Finance, 47* (2), 427–465.

Fama, E.F., & French, K.R. (1995). Size and book-to-market factors in earnings and returns. *Journal of Finance* (50), 131–155.

Fama, E.F., & French, K.R. (1996). Multifactor Explanations of Asset Pricing Anomalies. *The Journal of Finance, 51* (1), 55–84.

Fama, E.F., & French, K.R. (1998). Value versus Growth: The International Evidence. *The Journal of Finance, 53* (6), 1975–1999.

Fama, E.F., & French, K.R. (2004). The Capital Asset Pricing Model: Theory and Evidence. *Journal of Economic Perspectives, 18* (3), 25–46.

Fama, E.F., & French, K.R. (2015). A Five-factor Asset Pricing Model. *Journal of Financial Economics*, 1–22.

Fama, E.F., & MacBeth, J.D. (1973). Risk, Return, and Equilibrium: Empirical Tests. *The Journal of Poilitical Economy, 81* (3), 607–636.

Fama, E.F., & R., F.K. (1993). Common Risk Factors in the Returns on Stocks and Bonds. *Journal of Financial Economics, 33*, 3–56.

Fletcher, J. (2001). An Examination of Alternative Factor Models in UK Stock Returns. *Review of Quantitive Finance and Accounting* (16), 117–130.

Fletcher, J., & Forbes, D. (2002). U.K.Unit Trust Performance: Does It Matter which Benchmark or Measure Is Used? *Journal of Financial Services Research* (21), 195–218.

Fletcher, J., & Kihanda, J. (2005). An Examination of Alternative CAPM-based Models in UK Stock Returns. *Journal of Banking and Finance* (29), 2995–3014.

Foye, J., Mramor, D., & Pahor, M. (2013). A Respecified Fama French Three-Factor Model for the New European Union Member States. *Journal of International Financial Management & Accounting, 1* (24), 3–25.

Goriaev, A., & Zabotkin, A. (2006). Risks of Investing in the Russian Stock Market: Lessons of the First Decade. *Emerging Markets Review* (7), 380–397.

Gregory, A., Harris, R., & Michou, M. (2001). An Analysis of Contrarian Investment Strategies in the UK. *Journal of Business Finance and Accounting* (28), 1192–1228.

Guriev, S., & Tsyvinski, A. (2010). Challenges Facing the Russian Economy after the Crisis. In A.Aslund, S.Guriev, & A.Kuchins, *Russia after the Global Economic Crisis* (pp. 10–38). Washington D.C.: Peterson Institute for International Economics.

Hair, J.F., Black, W.C., Babin, B.J., & Anderson, R.E. (2010). *Multivariate data analysis: a global perspective* (7th ed.). London: Pearson.

Hassan, A., & Javed, M.T. (2011). Size and Value Premium in Pakistani Equity Market. *African Journal of Business Management, 5* (16), 6747–6755.

Hussain, S., Toms, J., & Diacon, S. (2002). Financial Distress, Market Anomalies and Single and Multifactor Asset Pricing Models: New Evidence. Jagadeesh, N., & Titman, S. (1993). Returns to buying winners and selling losers: Implications for stock market efficiency. *Journal of Finance* (48), 65–91.

Jensen, M.C. (1972). Capital Markets: Theory and Evidence. *Bell Journal of Economics and Management, 3* (2), 357–398. dKinnunen, J. (2012). Dynamic Return Predictability in the Russian Stock Market. *Emerging Markets Review* (15), 107–121.

Kothari, S.P., Shanken, J., & Sloan, R.G. (1995). Another Look at the Cross-Section of Expected Stock Returns. *Journal of Finance* (50), 185–224.

Kvint, V.L. (1998). *Emerging Market of Russia.* John Wiley & Sons, INC.

Lakonishok, J., Shleifer, A., & Vishny, R.W. (1994). Contrarian investment, extrapolation, and risk. *Journal of Finance* (49), 1541–1578.

Lam, K. (2005). Is the Fama-French Three-Factor Model Better Than The CAPM?

Lee, N., & Lings, I. (2008). *Doing business research: a guide to theory and practice.* London: SAGE.

Lintner, J. (1965). The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. *The Review of Economics and Statistics, 47* (1), 13–37.

Lui, W., Strong, N., & Xu, X. (1999). The Profitability of Momentum Investing. *Journal of Business Finance and Accounting* (26), 1043–1091.

MacKinlay, A.C. (1995). Multifactor Models do not explain deviations from the CAPM. *Journal of Financial Economics* (38), 3–28.

Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance, 7* (1), 77–91.

Michou, M., Mouselli, S., & Stark, A. (2007). *Estimating the Fama and French Factors in the UK — an Empirical Review.* Manchester.

Miles, D., & Timmermann, A. (1996). Variation in Expected Stock Returns: Evidence on the Pricing of Equities from A Cross-section of UK Companies. *Economica* (63), 369–382.

Miles, D., & Timmermann, A. (1996). Variation in expected stock returns: evidence on the pricing of equities from a cross-section of UK companies. *Economica*, 369–382.

Moody's. (2015, January 16). *Moody's downgrades Russia's government bond rating to Baa³; on review for further downgrade*. Retrieved March 17, 2015, from Moody's.

Moscow Exchange. (2015, March 12). *Moscow Exchange announces results for the full year 2014*. Retrieved March 17, 2015, from Moscow Exchange: http: //moex. com.

Mossin, J. (1966). Equilibrium in a Capital Asset Market. *Econometrica, 34* (4), 768–783.

New York Times. (1997, October 5). Bear Traps in the Russian Stock Market. *New York Times*.

Newbold, P., Carlson, W.L., & Thorne, B.M. (2010). *Statistics for Business and Economics* (7th ed.). London: Pearson.

Novy-Marx, R. (2013). The Other Side of Value: The gross Profitability Premium. *Journal of Financial Economics* (108), 411–433.

Peresetsky, A. (2014). What Drives the Russian Stock Market: World Market and Political Shocks. *Int.J.Computational Economics and Econometrics, 4* (1/2), 82–95.

Rafi, B., Kazmi, S.S., & Haslim, M. (2014). Validity of FAMA and French Model: Evidence from KSE-100 Index. *International*

Journal of Management Science, 1 (3), 52–57.
Rozhkov, A. (2005). Forecasting Russian Stock Market Trends. *Problems of Economic Transition, 48* (6), 48–65.
Saleem, K. (2014). Modeling Long Memory in the Russian Stock Market: Evodence from Major Sectoral Indicies. *The Journal of Applied Business Research, 30* (2), 567–574.
Saleem, K., & Vaihekovski, M. (2008). Pricing of Global and Local Source of Risk in Russian Stock Market. *Emerging Markets Review* (9), 40–56.
Saunders, M., Thornhill, A., & Lewis, P. (2012). *Research Methods for Business Students* (6th ed.). New York: Pearson.
Sharpe, W.F. (1964). Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. *The Journal of Finance, 19* (3), 425–442.
Sollogoub, T. (2003, October 14). *Moody's Lays a Bear Trap for Western Investors*. Retrieved March 17, 2015, from *The Moscow Times*
Soumare, I., Amenounve, E.K., Meite, D., & N'Sougan, Y.D. (2013). Applying CAPM and Fama-French models to the BRVM stock market. *Applied Financial Economics* (23), 275–285.
Tabachnick, B.G., & Fidell, L.S. (2013). *Using Multivariate Statistics* (6th ed.). London: Pearson.
Tadeo, M. (2014, November 17). *Apple now worth more than the entire Russian stock market*. Retrieved April 11, 2015, from *The Independent.*
The Economist. (2014, November 22). *The Russian economy: the End of the Line*. Retrieved April 11, 2015, from *The Economist*.
Titman, S., Wei, K., & Xie, F. (2004). Capital Investments and Stock Returns. *Journal of Financial and Quantitative Analysis* (39), 677–700.
Tverberg, G. (2014, December 7). *Ten Reasons Why a Severe Drop in Oil Prices is a Problem*. Retrieved 12 14, 2014, from *Our Finite World*.

APPENDICES

Appendix A.Companies on RTS index.

N	Code	Security name	Number of issued shares	Weight
1	GAZP	JSC „GAZPROM", Ordinary shares	23 673 512 900	15,00%
2	LKOH	OAO „LUKOIL", Ordinary shares	850 563 255	12,59%
3	SBER	Sberbank, Ordinary shares	21 586 948 000	10,51%
4	SBERP	Sberbank, Preferred shares	1 000 000 000	0,73%
5	MGNT	PJSC „Magnit", Ordinary shares	94 561 355	8,46%
6	SNGS	„Surgutneftegas" OJSC, Ordinary shares	35 725 994 705	3,66%
7	SNGSP	„Surgutneftegas" OJSC, Preferred shares	7 701 998 235	2,71%
8	GMKN	„OJSC „MMC „NORILSK NICKEL", Ordinary shares	158 245 476	5,88%
9	NVTK	JSC „NOVATEK", Ordinary shares	3 036 306 000	5,29%
10	ROSN	Rosneft, Ordinary shares	10 598 177 817	4,17%
11	MTSS	MTS OJSC, Ordinary shares	2 066 413 562	3,57%
12	VTBR	JSC VTB Bank, Ordinary shares	12 960 541 337 338	3,31%
13	TATN	JSC „TATNEFT", Ordinary shares	2 178 690 700	2,43%
14	TATNP	JSC „TATNEFT", Preferred shares	147 508 500	0,30%
15	TRNFP	JSC „Transneft", Preferred shares	1 554 875	2,60%
16	URKA	OJSC Uralkali, Ordinary shares	2 936 015 891	1,81%
17	POLY	Polymetal International plc, Ordinary shares	420 819 943	1,37%
18	YNDX	Yandex N.V., Ordinary shares	260 424 342	1,36%
19	MFON	OJSC „MegaFon", Ordinary shares	620 000 000	1,28%
20	RTKM	OJSC „Rostelecom", Ordinary shares	2 669 204 301	1,12%
21	RTKMP	OJSC „Rostelecom", Preferred shares	242 831 469	0,14%
22	ALRS	OJSC „ALROSA", Ordinary shares	7 364 965 630	1,13%
23	CHMF	OAO Severstal, Ordinary shares	837 718 660	1,12%

N	Code	Security name	Number of issued shares	Weight
24	HYDR	JSC „RusHydro", Ordinary shares	386 255 464 890	1,11%
25	MOEX	Moscow Exchange, Ordinary shares	2 278 636 493	0,98%
26	RUALR	Rusal, RDR	2 000 000 000	0,82%
27	NLMK	NLMK, Ordinary shares	5 993 227 240	0,71%
28	AFKS	Sistema JSFC, Ordinary shares	9 650 000 000	0,63%
29	PHOR	OJSC „PhosAgro", Ordinary shares	129 500 000	0,56%
30	PIKK	„PIK Group", Ordinary shares	660 497 344	0,53%
31	BANE	JSOC Bashneft, Ordinary shares	150 570 662	0,32%
32	BANEP	JSOC Bashneft, Preferred shares	29 788 012	0,22%
33	EONR	JSC „E.ON Russia", Ordinary shares	63 048 706 145	0,39%
34	MAGN	OJSC MMK, Ordinary shares	11 174 330 000	0,27%
35	LSRG	OJSC LSR Group, Ordinary shares	103 030 215	0,27%
36	DIXY	DIXY Group, Ordinary shares	124 750 000	0,25%
37	PHST	JSC „Pharmstandard", Ordinary shares	37 792 603	0,25%
38	GCHE	OJSC „Cherkizovo Group", Ordinary shares	43 963 773	0,24%
39	TRMK	TMK, Ordinary shares	937 586 094	0,24%
40	IRAO	JSC „Inter RAO", Ordinary shares	104 400 000 000*	0,24%
41	MVID	OJSC „Company „M. video", Ordinary shares	179 768 227	0,23%
42	AFLT	JSC „Aeroflot", Ordinary shares	1 110 616 299	0,21%
43	FEES	„FGC UES", JSC, Ordinary shares	1 274 665 323 063	0,20%
44	RSTI	JSC „ROSSETI", Ordinary shares	161 078 853 310	0,16%
45	AKRN	JSC Acron, Ordinary shares	40 534 000	0,13%
46	VSMO	VSMPO-AVISMA Corporation, Ordinary shares	11 529 538	0,11%
47	MSTT	OJSC „MOSTOTREST", Ordinary shares	282 215 500	0,10%
48	BSPB	„Bank „Saint-Petersburg" OJSC, Ordinary shares	439 554 000	0,09%
49	SVAV	SOLLERS OJSC, Ordinary shares	34 270 159	0,09%
50	NMTP	PJSC „NCSP", Ordinary shares	19 259 815 400	0,07%

Highlighted companies either financial or have no data available

Appendix B.Market Return Calculation.

Date	Last Price	RM	RF				
01.12.2008	631.89		Date	Last price	Rf/100	Rm-Rf	Rm-Rf*100
01.01.2009	535.04	−0.1532703	01.01.2009	12.575	0.12575	−0.27902	−27.9020348
01.02.2009	544.58	0.01783044	01.02.2009	12.693	0.12693	−0.1091	−10.9099557
01.03.2009	689.63	0.26635205	01.03.2009	12.796	0.12796	0.138392	13.83920511
01.04.2009	832.87	0.20770558	01.04.2009	10.677	0.10677	0.100936	10.09355813
01.05.2009	1087.59	0.30583404	01.05.2009	11.28	0.1128	0.193034	19.30340437
01.06.2009	987.02	−0.0924705	01.06.2009	11.297	0.11297	−0.20544	−20.5440508
01.07.2009	1017.47	0.03085044	01.07.2009	11.316	0.11316	−0.08231	−8.23095613

Date	Last Price	RM	Date	Last price	Rf/100	Rm-Rf	Rm-Rf*100
01.08.2009	1066.53	0.04821764	01.08.2009	11.515	0.11515	−0.06693	−6.69323621
01.09.2009	1254.52	0.17626321	01.09.2009	10.878	0.10878	0.067483	6.748320872
01.10.2009	1348.54	0.074945	01.10.2009	9.276	0.09276	−0.01782	−1.78150011
01.11.2009	1374.93	0.01956931	01.11.2009	9.087	0.09087	−0.0713	−7.1300688
01.12.2009	1444.61	0.05067894	01.12.2009	8.001	0.08001	−0.02933	−2.93310563
01.01.2010	1473.81	0.02021307	01.01.2010	7.751	0.07751	−0.0573	−5.72969321
01.02.2010	1410.85	−0.0427192	01.02.2010	7.723	0.07723	−0.11995	−11.9949211
01.03.2010	1572.48	0.11456214	01.03.2010	6.933	0.06933	0.045232	4.523214339
01.04.2010	1572.84	0.00022894	01.04.2010	7.113	0.07113	−0.0709	−7.09010623
01.05.2010	1384.59	−0.119688	01.05.2010	7.55	0.0755	−0.19519	−19.5187953
01.06.2010	1339.35	−0.0326739	01.06.2010	7.173	0.07173	−0.1044	−10.4403932
01.07.2010	1479.73	0.10481204	01.07.2010	7.077	0.07077	0.034042	3.404203569
01.08.2010	1421.21	−0.0395478	01.08.2010	7.308	0.07308	−0.11263	−11.2627755
01.09.2010	1507.66	0.06082845	01.09.2010	7.264	0.07264	−0.01181	−1.1811551
01.10.2010	1587.14	0.05271746	01.10.2010	7.591	0.07591	−0.02319	−2.31925438
01.11.2010	1597.35	0.00643295	01.11.2010	7.67	0.0767	−0.07027	−7.02670451
01.12.2010	1770.28	0.10826056	01.12.2010	7.441	0.07441	0.033851	3.385055655
01.01.2011	1870.31	0.05650519	01.01.2011	8.25	0.0825	−0.02599	−2.59948144
01.02.2011	1969.91	0.0532532	01.02.2011	8.763	0.08763	−0.03438	−3.4376796
01.03.2011	2044.2	0.03771238	01.03.2011	7.817	0.07817	−0.04046	−4.04576172
01.04.2011	2026.94	−0.0084434	01.04.2011	7.734	0.07734	−0.08578	−8.57834008
01.05.2011	1888.6	−0.0682507	01.05.2011	8.179	0.08179	−0.15004	−15.0040664
01.06.2011	1906.71	0.00958911	01.06.2011	8.127	0.08127	−0.07168	−7.16808864
01.07.2011	1965.02	0.03058147	01.07.2011	7.71	0.0771	−0.04652	−4.65185272
01.08.2011	1702.28	−0.1337086	01.08.2011	8.023	0.08023	−0.21394	−21.3938563
01.09.2011	1341.09	−0.2121801	01.09.2011	8.711	0.08711	−0.29929	−29.9290135
01.10.2011	1563.28	0.16567866	01.10.2011	8.712	0.08712	0.078559	7.855866437
01.11.2011	1540.81	−0.0143736	01.11.2011	8.306	0.08306	−0.09743	−9.74336247
01.12.2011	1381.87	−0.1031535	01.12.2011	8.5	0.085	−0.18815	−18.8153536
01.01.2012	1577.29	0.14141707	01.01.2012	8.29	0.0829	0.058517	5.851706528
01.02.2012	1734.99	0.09998161	01.02.2012	8.018	0.08018	0.019802	1.980161403
01.03.2012	1637.73	−0.056058	01.03.2012	7.825	0.07825	−0.13431	−13.430796
01.04.2012	1593.97	−0.0267199	01.04.2012	7.991	0.07991	−0.10663	−10.6629911
01.05.2012	1242.43	−0.2205437	01.05.2012	8.706	0.08706	−0.3076	−30.7603674
01.06.2012	1350.51	0.08699082	01.06.2012	8.46	0.0846	0.002391	0.239081638
01.07.2012	1377.35	0.01987397	01.07.2012	7.976	0.07976	−0.05989	−5.98860265
01.08.2012	1389.72	0.00898101	01.08.2012	7.86	0.0786	−0.06962	−6.96189857
01.09.2012	1475.7	0.06186858	01.09.2012	7.761	0.07761	−0.01574	−1.57414222
01.10.2012	1433.96	−0.0282849	01.10.2012	7.405	0.07405	−0.10233	−10.2334882

Date	Last Price	RM	Date	Last price	Rf/100	Rm-Rf	Rm-Rf*100
01.11.2012	1436.55	0.00180619	01.11.2012	6.93	0.0693	−0.06749	−6.74938129
01.12.2012	1526.98	0.06294943	01.12.2012	6.85	0.0685	−0.00555	−0.55505726
01.01.2013	1622.13	0.06231254	01.01.2013	6.6	0.066	−0.00369	−0.36874615
01.02.2013	1534.41	−0.054077	01.02.2013	6.69	0.0669	−0.12098	−12.0977047
01.03.2013	1460.04	−0.0484681	01.03.2013	6.91	0.0691	−0.11757	−11.7568141
01.04.2013	1407.21	−0.0361839	01.04.2013	6.53	0.0653	−0.10148	−10.148394
01.05.2013	1331.43	−0.0538512	01.05.2013	7.36	0.0736	−0.12745	−12.7451238
01.06.2013	1275.44	−0.0420525	01.06.2013	7.62	0.0762	−0.11825	−11.825253
01.07.2013	1313.38	0.0297466	01.07.2013	7.52	0.0752	−0.04545	−4.54534027
01.08.2013	1290.96	−0.0170705	01.08.2013	7.71	0.0771	−0.09417	−9.41704594
01.09.2013	1422.49	0.10188542	01.09.2013	7.31	0.0731	0.028785	2.87854186
01.10.2013	1480.42	0.04072436	01.10.2013	7.15	0.0715	−0.03078	−3.07756364
01.11.2013	1402.93	−0.0523433	01.11.2013	7.81	0.0781	−0.13044	−13.0443254
01.12.2013	1442.73	0.0283692	01.12.2013	7.71	0.0771	−0.04873	−4.87308013
01.01.2014	1301.02	−0.0982235	01.01.2014	8.39	0.0839	−0.18212	−18.2123507
01.02.2014	1267.27	−0.0259412	01.02.2014	8.33	0.0833	−0.10924	−10.9241185
01.03.2014	1226.1	−0.0324872	01.03.2014	8.93	0.0893	−0.12179	−12.1787157
01.04.2014	1155.7	−0.0574178	01.04.2014	9.47	0.0947	−0.15212	−15.2117829
01.05.2014	1295.75	0.12118197	01.05.2014	8.6	0.086	0.035182	3.518196764
01.06.2014	1366.08	0.05427745	01.06.2014	8.33	0.0833	−0.02902	−2.90225545
01.07.2014	1219.36	−0.1074022	01.07.2014	9.51	0.0951	−0.2025	−20.2502202
01.08.2014	1190.23	−0.0238896	01.08.2014	9.74	0.0974	−0.12129	−12.1289581
01.09.2014	1123.72	−0.05588	01.09.2014	9.4	0.094	−0.14988	−14.9879956
01.10.2014	1091.44	−0.028726	01.10.2014	9.99	0.0999	−0.12863	−12.8626017
01.11.2014	974.27	−0.1073536	01.11.2014	10.61	0.1061	−0.21345	−21.3453588
01.12.2014	790.71	−0.1884077	01.12.2014	14.09	0.1409	−0.32931	−32.9307731

Appendix C.Portfolio and Factor Returns.

Excess return		Risk free	S/L return	S/M return	S/H return	B/L return	B/M return	B/H return
2009	30.01.2009	12.575	−19.72821264	−36.33608022	−39.63294549	−31.77277103	−26.64923913	−25.05577618
	27.02.2009	12.693	−18.39041193	4.579412408	0.349685687	4.95747341	2.544722864	1.185818496
	31.03.2009	12.796	1.315738079	12.86562497	16.73473149	5.717032891	5.16589185	23.43613258
	30.04.2009	10.677	15.08008411	63.53633937	47.01308271	15.13763507	14.50655863	16.13842889
	29.05.2009	11.28	4.612088535	23.78438023	8.332628757	20.04380119	27.10423529	22.78314632
	30.06.2009	11.297	−17.36588399	−10.43592417	−16.21419471	−23.44142634	−20.83337368	−19.23381567
	31.07.2009	11.316	−13.36663461	−3.857581253	0.432711405	−10.09245713	5.806212528	−4.363674885
	31.08.2009	11.515	−15.22753552	−4.788282973	−3.635388716	−7.247312892	−9.091573987	−6.309312234
	30.09.2009	10.878	1.773348176	17.53228207	37.24081571	2.305959868	1.601230332	2.354461073

Excess return		Risk free	S/L return	S/M return	S/H return	B/L return	B/M return	B/H return
	30.10.2009	9.276	−0.035442071	−1.320629397	−5.392566218	−7.9492864	−4.755519262	−1.483822317
	30.11.2009	9.087	−4.323683104	−7.353102944	−1.743180162	−4.924419497	−2.130241278	−1.873876611
	31.12.2009	8.001	−6.750649848	7.141512134	−8.31324979	−10.35700775	−4.159408843	−5.193591357
2010	29.01.2010	7.75	−6.582647483	4.173696245	4.712763607	−2.645402063	1.829387862	−5.456748433
	27.02.2010	7.72	−8.379469746	2.43861495	2.065557229	−10.97655231	−8.883203277	−10.46151893
	31.03.2010	6.93	14.10495414	5.95439843	3.906278731	8.057580168	2.551152887	7.947338527
	30.04.2010	7.11	−15.86784496	−3.730333392	−6.053989687	−6.459929307	−2.379862855	−3.675050912
	31.05.2010	7.55	−24.6954747	−22.15952697	−22.30114889	−16.77378892	−24.307627	−16.08173204
	30.06.2010	7.17	−5.204209608	−12.84440984	−7.200113444	−12.53392766	−18.55303678	−11.35690502
	30.07.2010	7.08	5.54298619	0.206181764	0.357891874	4.84091959	5.685528362	6.610198107
	31.08.2010	7.31	−8.730019194	−8.755830532	−8.904793105	−11.92507029	−8.480712177	−6.808787908
	30.09.2010	7.26	1.734637958	−3.749141356	−3.814544943	−1.231758752	2.129519628	2.255290864
	29.10.2010	7.59	−2.095625172	−4.169306616	6.120528092	−4.561228091	−6.7800155	−4.655333547
	30.11.2010	7.67	−8.406649629	−4.928067145	−5.785114982	−9.249897876	−5.662728783	−3.383719248
	30.12.2010	7.44	0.253368382	−1.427168951	1.8376463	−1.776909391	5.31199093	14.14031085
2011	31.01.2011	8.25	−1.115675424	−1.359599207	−4.683049847	−2.378681046	−1.053944367	−7.37639929
	28.02.2011	8.76	−7.985943041	−6.921655063	−4.996170655	−4.763556382	−0.090813809	−2.815057466
	31.03.2011	7.82	−6.112985228	−9.374001829	−9.744698911	−7.62619745	−5.487235846	−2.27834988
	29.04.2011	7.73	−14.20409809	−9.568947775	−11.62311372	−11.27237176	−10.94161925	−7.269850951
	31.05.2011	8.18	−16.52602428	−11.93463922	−8.421294492	−15.21604092	−13.66392583	−8.120155438
	30.06.2011	8.13	−7.698123796	−7.143747116	−9.885751964	−3.806236668	−1.426014409	−8.752232838
	29.07.2011	7.71	−5.662759697	−4.384599961	−1.515304565	−4.510638582	−4.046844966	0.210661655
	31.08.2011	8.02	−25.29140002	−30.91359815	−20.08526508	−23.67965899	−22.21305997	−18.35276883
	30.09.2011	8.71	−35.91106769	−29.81133409	−19.1518306	−26.81526396	−36.02876397	−29.70281077
	31.10.2011	8.71	18.41952587	12.83114032	−3.541665205	8.636976255	15.41311715	6.80886365
	30.11.2011	8.31	0.799248896	−11.53515259	−8.650157015	−5.54308493	−7.866111869	−11.97966033
	30.12.2011	8.50	−22.3636752	−23.76036802	−21.34217125	−21.35030951	−18.82498953	−20.49725124
2012	31.01.2012	8.29	14.44928966	6.566822226	10.92971587	6.701004898	7.14429532	6.989660374
	29.02.2012	8.02	−0.969300785	−0.874474818	5.644644803	2.222767519	−2.571764648	1.155024408
	30.03.2012	7.83	−5.112824201	−7.903872667	−8.141262976	−15.76974014	−13.3690868	−10.62339149
	28.04.2012	7.99	−17.03807523	−8.574021187	−9.256441996	−14.08960302	−10.13641025	−8.157069678
	31.05.2012	8.71	−41.38997155	−29.60483607	−29.86443386	−31.01371812	−28.37141461	−25.71177302
	29.06.2012	8.46	4.969167269	−5.756621741	−3.537305053	1.194595226	−0.219215876	4.932396839
	31.07.2012	7.98	−4.308429444	0.060002265	−1.107207357	−6.480018911	−6.791346127	−2.98697694
	31.08.2012	7.86	−5.651843646	−8.000905746	−2.825683965	−9.079465547	−5.949423112	−10.1449125
	28.09.2012	7.76	12.31932669	−4.531940252	−0.200344121	−0.171929624	3.071849773	−4.612552012
	31.10.2012	7.41	−13.98676427	−10.09816867	−5.789588874	−12.07488584	−8.892853482	−7.964648364
	30.11.2012	6.93	−0.46376053	−4.219760012	−6.777685166	−6.468551005	−8.390720782	−6.84331812
	28.12.2012	6.85	−5.041150064	−1.285021858	5.46571189	−4.063452117	0.847178882	2.232372445

Excess return		Risk free	S/L return	S/M return	S/H return	B/L return	B/M return	B/H return
2013	31.01.2013	6.60	−0.552804291	5.635668586	−1.143369839	1.48212265	−3.332505902	−1.068430901
	28.02.2013	6.69	−17.96311588	−5.968661244	−9.116912806	−13.32328245	−13.77592715	−6.681903161
	29.03.2013	6.91	−20.12755309	−14.04139633	−12.38777747	−12.2444703	−14.81018082	−9.073992559
	30.04.2013	6.53	−25.77657545	−10.25160437	−8.863511747	−10.88489005	−9.250567231	−5.759191262
	31.05.2013	7.36	−11.02992481	−11.85679154	−2.850630309	−14.31556639	−12.540811	−8.428459749
	28.06.2013	7.62	−12.19203794	−7.570582757	−11.06004754	−9.348807742	−14.70718804	−4.226227649
	31.07.2013	7.52	−4.505961945	−8.437611937	−2.516414376	0.696308979	2.934592015	−4.447860065
	30.08.2013	7.71	−12.13615766	−10.80023978	−14.9455255	−8.389804046	−3.690474513	−10.09646232
	30.09.2013	7.31	−5.984026902	−3.383796048	−1.470341863	1.281752312	−3.027100409	4.215271861
	31.10.2013	7.15	−12.08145712	−2.783206249	−3.575804298	−3.41633146	−2.356212794	−2.111143017
	29.11.2013	7.81	−21.66433942	−13.76525126	−1.708370891	−13.19557616	−13.98985701	−11.41276352
	30.12.2013	7.71	3.188647883	−1.125006942	−5.070358172	−5.241958867	−4.519890774	−3.921838374
2014	31.01.2014	8.39	−15.98225971	−17.68566987	−29.38937671	−13.35744352	−17.94721508	−20.96411795
	28.02.2014	8.33	−19.50465795	−11.69134737	−5.255090106	−11.46010086	−10.467399	−5.925412063
	31.03.2014	8.93	−20.72174722	−13.21953684	−14.46743817	−8.912013286	−12.08504252	−14.65915682
	30.04.2014	9.47	−20.13617147	−5.335384344	−5.97424	−13.45053776	−14.30159712	−14.03541342
	30.05.2014	8.60	7.097977213	1.841096917	13.63315336	10.54860177	−2.867843711	6.784463341
	30.06.2014	8.33	−4.515921925	0.012443634	−6.663242964	−4.205746189	−0.611567816	−0.825278428
	31.07.2014	9.51	−20.04237094	−17.73863001	−16.89032658	−20.11551796	−16.50444922	−11.23486283
	29.08.2014	9.74	−10.66672734	−12.42346596	−9.616919633	−7.652314776	−8.144987191	−11.77036551
	30.09.2014	9.40	−17.80486826	−11.74676508	−8.352807762	−12.84270593	−12.58402409	−10.30214226
	31.10.2014	9.99	−17.41752488	−20.43896404	−7.988448766	−17.41813329	−11.95157592	−8.415636781
	28.11.2014	10.61	−22.3985549	−22.85012079	−15.72240212	−29.66080774	−20.31521596	−20.11267458
	30.12.2014	14.09	−45.28022728	−29.08015218	−38.16437407	−32.97312081	−31.22990192	−32.65832548

Appendix D.Factor Correlation

	SMB	HML
SMB	1	
HML	0.217556	1

Appendix E.Initial Regression Results

Initial Regression Results						
Fama-French					CAPM	
Factors	α_{it}	β_{im}	β_{is}	β_{ih}	α	β_{im}
S/L	1.1303***	1.0030***	0.7128***	– 0.8329***	−0.833	1.082***
S/M	1.3849***	1.0566***	0.9213***	0.2685***	2.819**	1.144***
S/H	0.1579	0.9752***	0.9293***	0.6837***	2.824**	1.058***
B/L	0.3083*	0.9942***	−0.0143	– 0.1600**	−0.174	0.995***

B/M	1.0841*	1.0186***	– 0.1915*	−0.0442	0.82	1.000***
B/H	1.2807*	1.0220***	– 0.2308***	0.3234***	2.075***	0.996***

*	Significant level of 10%
**	Significant level of 5%
***	Significant level of 1%

White's test		
Portfolio	**Fama-French**	**CAPM**
S/L	0.935433526	**4.23848778**
S/M	**10.6971544**	0.01362068
S/H	0.109362511	**6.51996791**
B/L	2.718457331	0.07874379
B/M	0.240396121	2.63387205
B/H	1.909519883	1.42075774
Critical chi-square value	**2.71**	

Breusch-Godfrey LM test		
	Fama-French	**CAPM**
S/L	0.557561036	0.40213137
S/M	1.784380886	0.36993566
S/H	2.850135314	**6.667736**
B/L	3.202873923	1.52200262
B/M	2.213281447	5.08838262
B/H	3.154990149	1.90950933
Chi-square	9.49 (5%)	5.99 (5%)
	13.28 (1%)	9.21 (1%)

Appendix F.Adjusted Regression Results

Adjusted Regression Results						
Fama-French					**CAPM**	
Factors	α_{it}	β_{im}	β_{is}	β_{ih}	α	β_{im}
S/L	1.1303***	1.0030***	0.7128***	– 0.8329***	– 2.039***	3.334***
S/M	−0.1960	0.8861***	1.1999***	0.2139***	2.819**	1.144***
S/H	0.1579	0.9752***	0.9293***	0.6837***	4.554***	1.117***
B/L	0.3083*	0.9942***	−0.0143	– 0.1600**	−0.174	0.995***
B/M	1.0841*	1.0186***	– 0.1915*	−0.0442	0.82	1.000***
B/H	1.2807*	1.0220***	– 0.2308***	0.3234***	2.075***	0.996***

*	Significant level of 10%
**	Significant level of 5%
***	Significant level of 1%

White's test		
Portfolio	**Fama-French**	**CAPM**
S/L	0.935433526	7.74837526
S/M*	1.169987113	0.01362068
S/H	0.109362511	6.51996791
B/L	2.70145733	0.07874379
B/M	0.240396121	2.63387205
B/H	1.909519883	1.42075774
Critical chi-square value	**2.71**	

Breusch-Godfrey LM test		
	Fama-French	**CAPM**
S/L	0.557561036	0.40213137
S/M	1.784380886	0.36993566
S/H	2.850135314	0.60786256
B/L	3.202873923	1.52200262
B/M	2.213281447	5.08838262
B/H	3.154990149	1.90950933
Chi-square	9.49 (5%)	5.99 (5%)
	13.28 (1%)	9.21 (1%)

Appendix G.Adjusted Regression Statistics

	R squared		F-Statistics		P-value	
Factors	**Fama-French**	**CAPM**	**Fama-French**	**CAPM**	**Fama-French**	**CAPM**
S/L	94%	75%	346.611	208.402	0.00000	0.00000
S/M	90%	43%	113.510	148.978	0.00000	0.00000
S/H	93%	72%	311.552	129.542	0.00000	0.00000
B/L	89%	88%	182.433	519.778	0.00000	0.00000
B/M	86%	85%	137.601	391.692	0.00000	0.00000
B/H	93%	90%	310.675	600.156	0.00000	0.00000
Average	91%	75%				

Financial Market Modeling with Quantum Neural Networks*

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Abstract. Econophysics has developed as a research field that applies the formalism of statistical mechanics and quantum mechanics to address economics and finance problems. The branch of econophysics that applies quantum theory to economics and finance is called quantum econophysics. In finance, quantum econophysics' contributions have ranged from option pricing to market dynamics modeling, behavioral finance and applications of game theory, integrating the empirical finding, from human decision analysis, that shows that nonlinear update rules in probabilities, leading to non-additive decision weights, can be computationally approached from quantum computation, with resulting quantum interference terms explaining the non-additive probabilities. The current work draws on these results to introduce new tools from quantum artificial intelligence, namely quantum artificial neural networks as a way to build and simulate financial market models with adaptive selection of trading rules, leading to turbulence and excess kurtosis in the returns distributions for a wide range of parameters.

Аннотация. Эконофизика сформировалась как исследовательская область, которая применяет понятия статистической механики и квантовой механики для исследования экономических и финансовых проблем. Раздел эконофизики, который применяет квантовую теорию к экономике и финансам, именуется квантовой эконофизикой. В финансовой сфере квантовая эконофизика используется в ряде областей — от оценки опционов до моделирования рыночной динамики. В данной работе вводятся новые инструменты из области квантового искусственного интеллекта, а именно квантовые искусственные нейронные сети в качестве способа создания адаптивных моделей финансовых рынков.

Key words: Finance, econophysics, quantum artificial neural networks, quantum stochastic processes, cognitive science.

1. INTRODUCTION

One of the major problems of financial modeling has been to address complex financial returns dynamics, in particular, excess kurtosis and volatility-related turbulence which lead to statistically significant deviations from the Gaussian random walk model worked in traditional Financial Theory (Arthur *et al.*, 1997; Voit, 2001; Ilinski, 2001; Focardi and Fabozzi, 2004). A main contribution of econophysics to finance has been to address this problem using the tools from statistical mechanics and quantum mechanics, within the paradigmatic basis of systems science and complexity sciences (Anderson *et al.*, 1988; Arthur *et al.*, 1997; Voit, 2001; Ehrentreich, 2008).

Econophysics is currently a major research area that has combined interdisciplinary finance and economics, complex systems science, statistical mechanics, quantum mechanics and cognitive science to address notions and problems in economics and finance (Anderson *et al.*, 1988; Arthur *et al.*, 1997; Voit, 2001; Brunn, 2006; Ehrentreich, 2008; Piotrowski and Sładkowski, 2001, 2002, 2008; Saptsin and Soloviev, 2009, 2011).

There are two major branches in econophysics: classical econophysics (based on classical mechanics) and quantum econophysics (based on quantum mechanics). In finance, quantum econophysics has been applied to option pricing (Segal and Segal, 1998; Baaquie *et al.*, 2000; Baaquie and Marakani, 2001; Baaquie, 2004; Baaquie and Pan, 2011), financial turbulence modeling (Gonçalves, 2011, 2013) and as an approach to the formulation of financial theory, regarding price formation and basic market relations (Piotrowski and Sładkowski, 2001, 2002, 2008; Khrennikov, 2010; Haven and Khrennikov, 2013; Gonçalves, 2011, 2013). Choustova (2007a, b), in particular, argued for the introduction of a quantum-based

* Моделирование финансовых рынков с использованием квантовых нейронных сетей.

approach to financial theory as a way to incorporate market cognition dynamics in financial price formation.

The quantum-based approach goes, however, beyond a good match to price dynamics and turbulence modeling. The growing empirical evidence of quantum interference signatures in human cognition, when faced with decision problems, has led to the development of a quantum theory-based cognitive science forming a theoretical ground for econophysics modeling, with strong implications for finance (Busemeyer and Franco, 2010; Busemeyer and Bruza, 2012; Wang and Busemeyer, 2013; Busemeyer and Wang, 2014; Khrennikov, 2010; Haven and Khrennikov, 2013; Zuo, 2014; Khrennikov and Basieva, 2014).

The main research problem regarding Quantum Theory-based Cognitive Science applied to Finance can be expressed as follows: *if* there is empirical support to the fact that human cognition, in decision problems, leads to a decision behavior computationally isomorphic to quantum adaptive computation (Busemeyer and Franco, 2010; Busemeyer and Bruza, 2012; Wang and Busemeyer, 2013; Busemeyer and Wang, 2014; Zuo, 2014; Khrennikov and Basieva, 2014; Gonçalves, 2015), *then*, the modeling of financial market dynamics needs to work with models of behavior that incorporate, in their probabilistic description, quantum interference terms (Khrennikov, 2010; Haven and Khrennikov, 2013).

This main research problem has led to the growth and development of research lines on cognitive science, working from quantum computer science and quantum information theory, with direct implications for finance and economics, supporting the expansion of quantum econophysics (Khrennikov, 2010; Haven and Khrennikov, 2013), in particular, in regards to finance: opening up the way for research on quantum artificial intelligence (QuAI) applications to financial market modeling (Gonçalves, 2011, 2013).

The current work contributes to such research by introducing Quantum Artificial Neural Networks (QuANNs) for financial market dynamics and volatility risk modeling. In particular, recurrent QuANNs are used to build a model of financial market dynamics that incorporates quantum interference and quantum adaptive computation in the probabilistic description of financial returns. The resulting model shows a quantum-based selection of adaptive rules with consequences for the market dynamics, leading to excess kurtosis and turbulence with clustering volatility, price jumps and statistically significant deviations from Gaussian distributions, for a wide range of parameters.

The work is divided in two parts that are developed in sections 2 and 3. In section 2, the QuANN model is built, simulated and studied, while, in section 3, a reflection is provided on the possible role and contributions of QuAI applied to financial modeling. Regarding the main work, which is developed in section 2, the structure of this section is divided in three subsections.

In subsection 2.1, we review a general framework for classical econophysics modeling of financial market price formation in which Farmer's market making model (Farmer, 2002) is reviewed and combined with multiplicative components, namely: multiplicative volatility components and a market polarization component are introduced in the market making model and linked to trading volume and bullish versus bearish polarization.

In subsection 2.2, we introduce the general formalism of QuANNs, including main notions that form the groundwork for the financial market model. In subsection 2.3, we build the financial market model using a Quantum Neural Automaton (QNA) structure and simulate the resulting artificial financial market, addressing its main results in regards to turbulence and volatility risk, leading to statistically significant deviations from the Gaussian returns distribution.

In section 3, the problem of deviations from the Gaussian random walk is addressed in its relation to econophysics and nonlinear stochastic models of market dynamics, allowing for a reflection on the possible contributions of QuAI and QuANNs for establishing a bridge between the evidence of quantum interference patterns observed in human decision making and a computational basis for nonlinear probability dynamics in finance coming from a linear unitary evolution of networked quantum computation.

2. A QUANN-BASED FINANCIAL MARKET MODEL

2.1 PRICE FORMATION AND FINANCIAL RETURNS

Following Farmer (2002) and Ilinski (2001), financial market price formation can be linked to unbalanced market orders M, where $M > 0$ corresponds to an excess demand while $M < 0$ to an excess supply, such that, for a financial risky asset, traded at discrete trading rounds of duration Δt, the asset price at t, $S(t)$ depends upon the previous price $S(t-\Delta t)$ and the market orders that arrive during the trading round. A few basic assumptions, in classical econophysics, determine the structure for the relation between market orders and the new price (Farmer, 2002; Ilinski, 2001):

The price is assumed as a finite increasing function of the previous price and order size $M(t)$:

$$S(t) = f_S\left(S(t-\Delta t), M(t)\right) \tag{1}$$

If the order size is null $M(t) = 0$ the market clears for equal supply and demand, so that there is no market impact (the price stays unchanged):

$$f_S\left(S(t-\Delta t),0\right) = S(t-\Delta t) \quad (2)$$

There are no arbitrage opportunities associated with a sequence of trades that sum zero (a repeated trading through a circuit);

Gauge invariance with respect to currency units, so that the only possible combination for prices to enter is $S(t)/S(t-\Delta t)$, such that:

$$\frac{S(t)}{S(t-\Delta t)} = \frac{f_S\left(S(t-\Delta t), M(t)\right)}{S(t-\Delta t)} = F\left(M(t)\right) \quad (3)$$

The result of these four assumptions is the general form for F in Eq. (3) given by (Farmer, 2002; Ilinski, 2001):

$$F\left(M(t)\right) = e^{\frac{M(t)}{\lambda}} \quad (4)$$

where λ is a liquidity parameter, also called market depth (Farmer, 2002). The result from Eq. (4), replaced in Eq. (3) is the following dynamical rule:

$$S(t) = S(t-\Delta t)e^{\frac{M(t)}{\lambda}} \quad (5)$$

or, taking the logarithms, the log-price rule (Farmer, 2002; Ilinski, 2001):

$$\ln S(t) = \ln S(t-\Delta t) + \frac{M(t)}{\lambda} \quad (6)$$

There are two dynamical components to $M(t)$: the sign, which can either be positive (excess of buy orders) or negative (excess of sell orders), and the volume of unbalanced market orders, which is linked to the order size.

Within financial theory, the order size can be worked from a systemic market dynamics that leads to the formation of consensus clusters regarding the decision to invest greater or smaller amounts, or, alternatively, to sell greater or smaller amounts.

The adaptive management of exposure to asset price fluctuation risk, on the part of market agents, given information that impacts asset value leads to a two-sided aspect of computation of financial information by the market system: on the one hand, there is the matter whether each new information is good (*bullish*) or bad (*bearish*), in terms of asset value, on the other hand, there is the degree to which new information supports the decision to buy or sell by different amounts (the market volume aspect).

A social consensus dynamics coming from market computation can be linked to consensus clusters affecting the market unbalance, so that the positive or negative sign can be addressed, within econophysics, in terms of a notion of spin. In physics the spin is a fundamental degree of freedom of field quanta that behaves like angular momentum, the spin quantum numbers assume integer and half-integer values, the most elementary case of half integer spin is the spin-1/2.

Considering a three dimensional axes system, if a spin-1/2 particle's spin state is measured along the z-axis then there are two fundamental orientations *spin up* and *spin down*, in complex systems science these two orientations are assumed and worked mainly from the statistical mechanics of Ising systems as models of complex systems (Kauffman, 1993), which constituted early inspiration for econophysics' models of financial markets (Vaga, 1990; Iiori, 1999; Lux and Marchesi, 1999; Voit, 2001). These models allowed for the study of polarization in market sentiment, working with the statistical mechanics of Ising systems, allowing direct connections to cognitive science (Voit, 2001).

The market volume, on the other hand, has been addressed, within financial theory, by multiplicative processes (Mandelbrot, *et al.*, 1997; Mandelbrot, 1997), drawing upon Mandelbrot's work on turbulence in statistical mechanics, as reviewed in Mandelbrot (1997). The multiplicative stochastic processes, worked by Mandelbrot and connected to multifractal geometry, led to Mandelbrot *et al.* 's (1997) Multifractal Model of Asset Returns (MMAR), which also inspired modified versions using multiplicative stochastic processes with Markov switching in volatility components (Calvet and Fisher, 2004; Lux, 2008).

Considering Eq. (6), a spin-1/2 like model can be integrated as a binary component in a multiplicative model that includes market volume, by way of a multiplicative decomposition of $M(t)$ in a market polarization component $\sigma(t) = \pm 1$, and N trading volume-related volatility components, so that we obtain:

$$M(t) = \left(\prod_{k=1}^{N} V_k(t)\right)\sigma(t) \quad (7)$$

where each volatility component $V_k(t)$ can assume one of two values v_0 or $v_1 = 2 - v_0$. If $0 < v_0 \leq 1$, then v_0 corresponds to a low volatility state, while v_1 to a high volatility state (v_0 diminishes the returns' value while v_1 amplifies the returns value like a lever). The logarithmic returns for the risky asset, in this approach, are given by:

$$R(t) = \ln \frac{S(t)}{S(t-\Delta t)} = \frac{1}{\lambda}\left(\prod_{k=1}^{N} V_k(t)\right)\sigma(t) \quad (8)$$

The binary structure assumed for the N components plus the market polarization, makes this model a good starting point for QuANN applications, since QuANNs also work from a binary computational basis to address neural firing patterns.

On the other hand, QuANNs open up the possibility for dealing with the multiplicative models in such a way that the probabilities, rather than being introduced from a top-down ex-ante fixed state-transition probability distribution, change from trading round to trading round, being the result of the quantum computational process introduced for each returns' component.

QuANNs also allow one to incorporate the empirical evidence that human cognition, when addressing decision between alternatives, follows a dynamics that is computationally isomorphic to quantum computation applied to decision science, leading to interference effects with an expression in decision frequencies (probabilities), which means that, when considering probabilities for human behavior, the theoretical framework of networked quantum computation may be more appropriate for the dynamical modeling of human systems.

In the quantum description, Eq. (8) will be expressed in operator form on an appropriate Hilbert space, with the returns operator's eigenvalues being addressed from the QuANN structure, which works with *quantum bits* (*qubits*), whose computational basis states describe the neuron's firing pattern in terms of firing (ON) and non-firing (OFF)[1]. In order to build the market model, however, we need to introduce, first, a general framework for QuANNs which will then be applied to the risky asset price dynamics modeling.

2.2 QUANTUM ARTIFICIAL NEURAL NETWORKS

The connection between quantum computer science and ANNs has been object of research since the 1990s, in particular, in what regards quantum associative memory, quantum parallel processing, extension of classical ANN schemes, as well as computational complexity and efficiency of QuANNs over classical ANNs (Chrisley, 1995; Kak, 1995; Menneer and Narayanan, 1995; Behrman *et al.*, 1996; Menneer, 1998; Ivancevic and Ivancevic, 2010; Gonçalves, 2015).

Mathematically, a classical ANN with a binary firing pattern can be defined as an artificial networked computing system comprised of a directed graph (digraph) with the following additional structure (McCulloch and Pitts, 1943; Müller *et al.*, 1995):

- A binary alphabet $A_2 = \{0,1\}$ associated to each neuron describing the neural activity, with 0 corresponding to a non-firing neural state and 1 to a firing neural state, so that the firing patterns of a neural network with N neurons are expressed by the set of all binary strings of length N: $A_2^N = \{s_1 s_2 \ldots s_N : s_k \in A_2, k = 1,2,\ldots,N\}$;
- A real-valued weight associated with each neural link, expressing the strength and type of neural connection;
- A transfer function which determines the state transition of the neuron and that depends upon: the state of its incident neurons, the weight associated with each incoming neural links and an activation threshold that can be specific for each neuron.

A quantum version of ANNs, on the other hand, can be defined as a directed graph with a networked quantum computing structure, such that (Gonçalves, 2015):

- To each neuron is associated a two-dimensional Hilbert Space H_2 spanned by the computational basis $B_2 = \{|0\rangle, |1\rangle\}$, where $|0\rangle, |1\rangle$ are ket vectors (in Dirac's *bra-ket* notation for Quantum Mechanics' vector-based formalism using Hilbert spaces[2]), where $|0\rangle$ encodes a non-firing neural dynamics and $|1\rangle$ encodes a firing neural dynamics;
- To a neural network, comprised of N neurons, is associated the tensor product of N copies of H_2, so that the neural network's Hilbert space is the space $H_2^{\otimes N}$ spanned by the basis $B_2^{\otimes N} = \{|\mathbf{s}\rangle : \mathbf{s} \in A_2^N\}$ which encodes all the alternative firing patterns of the neurons;
- The general neural configuration state of the neural network is characterized by a normalized ket vector $|\psi\rangle \in H_2^{\otimes N}$ expanded in the neural firing patterns' basis $B_2^{\otimes N}$:

$$|\psi\rangle = \sum_{\mathbf{s} \in A_2^N} \psi(\mathbf{s}) |\mathbf{s}\rangle \quad (9)$$

with the normalization condition:

$$\sum_{\mathbf{s} \in A_2^N} |\psi(\mathbf{s})|^2 = 1 \quad (10)$$

[1] This degree of freedom behaves like spin, so that the neuron's associated qubit can also be approached in terms of a spin-1/2 model.

[2] We use the vector representation convention introduced by Dirac (1967) for Hilbert spaces, assumed and used extensively in Quantum Mechanics. In this case, a *ket vector*, represented as $|a\rangle$, is a column vector of complex numbers while a *bra vector*, represented as $\langle a|$, is the conjugate transpose of $|a\rangle$, that is: $\langle a| = |a\rangle^\dagger$. The Hilbert space inner product is represented as $(|a\rangle, |b\rangle) = \langle a|b\rangle$. The outer product is, in turn, given by $|a\rangle\langle b|$. A projection operator corresponds to an operator of the form $\hat{P}_a = |a\rangle\langle a|$ which acts on any ket $|b\rangle$ as $\hat{P}_a|b\rangle = \langle a|b\rangle|a\rangle$.

The neural network has an associated neural links state transition operator $\hat{L}_{Net}$ such that, given an input neural state $|\psi_{in}\rangle$, the operator transforms the input state for the neural network in an output state $|\psi_{out}\rangle$, reflecting, in this operation, the neural links for the neural network, so that each neuron has an associated structure of unitary operators that is conditional on its input neurons:

$$|\psi_{out}\rangle = \hat{L}_{Net}|\psi_{in}\rangle \tag{11}$$

The output state of a QuANN shows, in general, complex quantum correlations so that the quantum dynamics of a single neuron may depend in a complex way on the entire neural network's configuration (Gonçalves, 2015). Considering the neurons n_1, ..., n_N for a N-neuron neural network, the $\hat{L}_{Net}$ operator can be expressed as a product of each neuron's neural links operator following the ordered sequence $n_1, ..., n_N$, where neuron n_1 is the first to be updated and n_N the last (that is, following the activation sequence[3]):

$$\hat{L}_{Net} = \hat{L}_N ... \hat{L}_2 \hat{L}_1 \tag{12}$$

Each neuron's neural links operator is a quantum generalization of an activation function, with the following structure for the k-th neuron:

$$\hat{L}_k = \sum_{\mathbf{s}\in A_2^{k-1}, \mathbf{s}'\in A_2^{N-k}} |\mathbf{s}\rangle\langle\mathbf{s}| \otimes L_k(\mathbf{s}_{in}) \otimes |\mathbf{s}'\rangle\langle\mathbf{s}'| \tag{13}$$

where $\mathbf{s}_{in}$ is a substring, taken from the binary word $\mathbf{ss}'$, that matches in $\mathbf{ss}'$ the activation pattern for the input neurons of n_k, under the neural network's architecture, in the same order and binary sequence as it appears in $\mathbf{ss}'$, $L_k(\mathbf{s}_{in})$ is a neural links function that maps the input substring to a unitary operator on the two-dimensional Hilbert space H_2, this means that, for different configurations of the neural network, the neural links operator for the k-th neuron $\hat{L}_k$ assigns a corresponding unitary operator that depends upon the activation pattern of the input neurons.

The neural links operators incorporate the local structure of neural connections so that there is a unitary state transition for the neuron (a quantum computation) conditional upon the firing pattern of its input neurons.

[3] For some QuANNs it is possible to consider the action of the operators conjointly and to introduce, in one single neural links operator, a transformation of multiple neurons' states, taking advantage of parallel quantum computation (Gonçalves, 2015).

Now, an arbitrary unitary operator on a single-*qubit* Hilbert space H_2 is a member of the unitary group U (2) and can be derived from a specific Hamiltonian operator structure (Greiner and Müller, 2001), so that we have, for a QuANN, a conditional unitary state transition:

$$L_k(\mathbf{s}_{in}) = e^{-\frac{i}{\hbar}\Delta t \hat{H}_{\mathbf{s}_{in}}} \tag{14}$$

where the neuron's associated Hamiltonian operator $\hat{H}_{\mathbf{s}_{in}}$ is conditional on the input neurons' firing pattern $\mathbf{s}_{in}$ and given by the general structure:

$$\hat{H}_{\mathbf{s}_{in}} = -\frac{\omega(\mathbf{s}_{in})}{2}\hbar\hat{1} + \theta(\mathbf{s}_{in})\sum_{j=1}^{3} u_j(\mathbf{s}_{in})\frac{\hbar}{2}\hat{\sigma}_j \tag{15}$$

where ħ is the reduced Planck constant[4], $\theta(\mathbf{s}_{in})$, $\omega(\mathbf{s}_{in})$ are measured in radians per second and depend upon the neural configuration for the input neurons, $\hat{1}$ is the unit operator on H_2, the $u_j(\mathbf{s}_{in})$ terms are the components of a real unit vector $\mathbf{u}(\mathbf{s}_{in})$ and $\hat{\sigma}_j$ are Pauli's operators[5]:

$$\hat{\sigma}_1 = |0\rangle\langle 1| + |1\rangle\langle 0| = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{16}$$

$$\hat{\sigma}_2 = -i|0\rangle\langle 1| + i|1\rangle\langle 0| = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{17}$$

$$\hat{\sigma}_3 = |0\rangle\langle 0| - |1\rangle\langle 1| = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{18}$$

Replacing Eq. (15) in Eq. (14) and expanding we obtain:

$$L_k(\mathbf{s}_{in}) = e^{-\frac{i}{\hbar}\Delta t \hat{H}_{\mathbf{s}_{in}}} = e^{i\frac{\omega(\mathbf{s}_{in})\Delta t}{2}} \left[\cos\left(\frac{\theta(\mathbf{s}_{in})\Delta t}{2}\right)\hat{1} - i\sin\left(\frac{\theta(\mathbf{s}_{in})\Delta t}{2}\right)\sum_{j=1}^{3} u_j(\mathbf{s}_{in})\hat{\sigma}_j \right] \tag{19}$$

The operator in Eq. (19) is comprised of the product of a phase transformation ($i\omega(\mathbf{s}_{in})\Delta t/2$) and a rotation operator defined as (Greiner and Müller, 2001; Nielsen and Chuang, 2003):

[4] $1.054571800(13)\times 10^{-34}$ Js

[5] The terms $(\hbar/2)\hat{\sigma}_j$, in the Hamiltonian, are equivalent to the spin operators for a spin-1/2 system (Leggett, 2002).

$$\hat{R}_{\mathbf{u}(\mathbf{s}_{in})}\left[\theta(\mathbf{s}_{in}),\Delta t\right]=\cos\left(\frac{\theta(\mathbf{s}_{in})\Delta t}{2}\right)\hat{1}-i\sin\left(\frac{\theta(\mathbf{s}_{in})\Delta t}{2}\right)\sum_{j=1}^{3}u_j(\mathbf{s}_{in})\hat{\sigma}_j \qquad (20)$$

An arbitrary single-*qubit* unitary operator (a quantum logic gate on a *qubit*) can, thus, be expressed by the product (Nielsen and Chuang, 2003):

$$e^{-\frac{i}{\hbar}\hat{H}_{\mathbf{s}_{in}}\Delta t}=\exp\left(i\frac{\omega(\mathbf{s}_{in})\Delta t}{2}\right)\hat{R}_{\mathbf{u}(\mathbf{s}_{in})}\left[\theta(\mathbf{s}_{in})\Delta t\right] \qquad (21)$$

This means that the transfer function of classical ANNs is replaced, for QuANNs, by phase transformations and rotations of the neuron's quantum state conditional upon the firing pattern of the input neurons[6].

Now, given an operator $\hat{O}$ on the neural network's Hilbert space $H_2^{\otimes N}$ expanded as:

$$\hat{O}=\sum_{\mathbf{s},\mathbf{s}'\in A_2^N}O_{\mathbf{s},\mathbf{s}'}\left|\mathbf{s}\right\rangle\left\langle\mathbf{s}'\right| \qquad (22)$$

taking the inner product between a normalized ket vector $\left|\psi\right\rangle$ and the transformed vector $\hat{O}\left|\psi\right\rangle$ yields:

$$\left(\left|\psi\right\rangle,\hat{O}\left|\psi\right\rangle\right)=\left\langle\psi\left|\hat{O}\right|\psi\right\rangle=\sum_{\mathbf{s},\mathbf{s}'\in A_2^N}O_{\mathbf{s},\mathbf{s}'}\left\langle\mathbf{s}'\mid\psi\right\rangle\left\langle\psi\mid\mathbf{s}\right\rangle=\sum_{\mathbf{s},\mathbf{s}'\in A_2^N}O_{\mathbf{s},\mathbf{s}'}\psi(\mathbf{s}')\psi(\mathbf{s})^* \qquad (23)$$

For Hermitian operators obeying the relation:

$$O_{\mathbf{s},\mathbf{s}'}\left\langle\mathbf{s}'\mid\psi\right\rangle\left\langle\psi\mid\mathbf{s}\right\rangle\delta_{\mathbf{s},\mathbf{s}'} \qquad (24)$$

given that the state vector is normalized, if this relation is verified, then Eq. (23) yields a classical expectation in which the amplitudes in square modulus $|\psi(\mathbf{s})|^2$ are equivalent to decision weights associated with each alternative value on the diagonal of the operator's matrix representation:

$$\left\langle\hat{O}\right\rangle_{\psi}=\left\langle\psi\left|\hat{O}\right|\psi\right\rangle=\sum_{\mathbf{s}\in A_2^N}O_{\mathbf{s},\mathbf{s}}\,|\psi(\mathbf{s})|^2 \qquad (25)$$

so that, for a neural network in the state $\left|\psi\right\rangle$, the neural activity can be described by the value $O_{\mathbf{s},\mathbf{s}}$ with an associated weight of $|\psi(\mathbf{s})|^2$.

In the case of econophysics, as well as game theory applications, one usually assumes that the social system tends to the alternatives in proportion to the corresponding decision weights, such that one can associate a probability measure for the system to follow each alternative as numerically coincident to the corresponding decision weight. This is akin to game theory's notion of mixed strategy, in the sense that each player can be characterized by a fixed mixed strategy and play probabilistically according to the mixed strategy's weights.

While the probability of a player's behavior is zero or one after play, the decision weights remain the same, in the case of game theory this means that the Nash equilibrium does not change, being available as a cognitive strategic scheme for further plays (Nash, 1951). In applications of QuANNs to social systems this means that one needs to work with either an Everettian interpretation of quantum theory, or with a Bohmian interpretation[7].

The Bohmian interpretation is often assumed by researchers dealing with econophysics (Choustova, 2007a, b; Khrennikov, 2010; Haven and Khrennikov, 2013), in particular, when one wishes to address the amplitudes in square modulus $|\psi(\mathbf{s})|^2$ in terms of economic forces linked to emergent degrees of freedom that tend to make the system follow certain paths probabilistically (a quantum-based probabilistic version of Haken's slaving principle applied to economic and financial systems (Haken, 1977)).

The Everettian line of interpretations has, since its initial proposal by Everett (1957, 1973), been directly linked to a Cybernetics' paradigmatic basis incorporating both automata theory and information theory

[6] This leads to quantum correlations that reflect the neural network's structure (Gonçalves, 2015).
[7] Since these are the two lines of interpretation that do not assume a state vector collapse.

(Gonçalves, 2015), a point that comes directly from Everett's original work on quantum mechanics, that is further deepened by Deutsch's work on quantum computation (Deutsch, 1985), and, later, on quantum decision theory (Deutsch, 1999; Wallace, 2002, 2007).

There are actually different perspectives from different authors on Everett's original proposal (Bruce, 2004). Formally, the proposal is close to Bohm's, including the importance attributed to computation and to information theory, however, systemically, Bohm and Everett are very distinct in the hypotheses they raise: for Bohm the state vector is assumed to represent a statistical average of an underlying information field's sub-quantum dynamics (Bohm, 1984; Bohm and Hiley, 1993), Everett (1957, 1973) assumes the geometry of the Hilbert space as the correct description of the fundamental dynamics of fields and systems.

Considering QuANNs, under Everett's approach, we can introduce the set of projection operators onto the basis $B_2^{\otimes N}$, $P = \left\{ \hat{P}_{\mathbf{s}} = |\mathbf{s}\rangle\langle\mathbf{s}| : \mathbf{s} \in A_2^N \right\}$ where each operator has the matrix representation $P_{\mathbf{s},\mathbf{s}'} = \delta_{\mathbf{s},\mathbf{s}'}$, these operators form a complete set of orthogonal projectors, since their sum equals the unit operator on the Hilbert space $H_2^{\otimes N}$, $\sum_{\mathbf{s}\in A_2^N} \hat{P}_{\mathbf{s}} = \hat{1}^{\otimes N}$, and they are mutually exclusive, that is, the product of two of these operators obeys the relation $\delta_{\mathbf{s},\mathbf{s}'} \hat{P}_{\mathbf{s}} \hat{P}_{\mathbf{s}'}$.

A projection operator can represent a projective computation, by the neural network, of an alternative neural firing pattern for the network. The general state vector in Eq. (9) can, thus, be expressed as a sum of projections, that is, the neural network's quantum state has a projective expression over each alternative neural configuration simultaneously, corresponding to a simultaneous systemic projective activity over all alternatives:

$$|\psi\rangle = \sum_{\mathbf{s}\in A_2^N} \hat{P}_{\mathbf{s}} |\psi\rangle \tag{26}$$

Each alternative neural configuration corresponds to an orthogonal dimension of the 2^N dimensional Hilbert space $H_2^{\otimes N}$, a dimension that is spanned by a corresponding basis vector in $B_2^{\otimes N}$, which means that the quantum system (in our case, the QuANN) projects simultaneously over each (orthogonal) dimension of systemic activity (corresponding, in our case, to each alternative neural pattern) weighing each dimension. The weight of the projection over a given dimension (a given pattern of systemic activity) in the system's state can be worked from a notion of norm. Using the Hilbert space's inner product structure, we can work with the squared norm of the projected vector, which leads to:

$$\begin{aligned} \left\| \hat{P}_{\mathbf{s}} |\psi\rangle \right\|^2 &= (\hat{P}_{\mathbf{s}}|\psi\rangle, \hat{P}_{\mathbf{s}}|\psi\rangle) = \\ &= \left\langle \psi \left| \hat{P}_{\mathbf{s}}^{\dagger} \hat{P}_{\mathbf{s}} \right| \psi \right\rangle = |\psi(\mathbf{s})|^2 \end{aligned} \tag{27}$$

Systemically, this last equation can be interpreted as expressing that the weight of the projection $\hat{P}_{\mathbf{s}}$, in the system's projective dynamics, is equal to $|\psi(\mathbf{s})|^2$. In this sense, each orthogonal dimension corresponds to a distinct pattern of activity that is projectively computed by the system.

On the other hand, for a large ensemble of QuANNs with the same structure and in the same state, the statistical weight associated to the projection operator $\hat{P}_{\mathbf{s}}$, expressed by the ensemble average $\left\langle \hat{P}_{\mathbf{s}} \right\rangle$, coincides with the projection weight $|\psi(\mathbf{s})|^2$ associated to the neural state projection $\hat{P}_{\mathbf{s}}|\psi\rangle$, thus, the statistical interpretation comes directly from the projective structure of the state vector. Indeed, let us consider a statistical ensemble of M QuANNs such that each QuANN has the same number of neurons N and the same architecture, let us, further, assume that each neural network is characterized by some quantum neural state $|\psi_k\rangle$, with $k = 1,2, \ldots, M$, the ensemble state can be represented by a statistical density operator:

$$\hat{\rho} = \frac{1}{M} \sum_{k=1}^{M} |\psi_k\rangle\langle\psi_k| \tag{28}$$

The statistical average of an operator $\hat{O}$ on the Hilbert space $H_2^{\otimes N}$ is given by (Bransden and Joachain, 2000):

$$\begin{aligned} \left\langle \hat{O} \right\rangle &= Tr(\hat{O}\hat{\rho}) = \\ &= \frac{1}{M} \sum_{k=1}^{M} \sum_{\mathbf{s},\mathbf{s}'\in A_2^N} O_{\mathbf{s},\mathbf{s}'} \langle \mathbf{s}' | \psi_k \rangle \langle \psi_k | \mathbf{s} \rangle \\ &= \frac{1}{M} \sum_{k=1}^{M} \left\langle \psi_k \left| \hat{O} \right| \psi_k \right\rangle = \frac{1}{M} \sum_{k=1}^{M} \left\langle \hat{O} \right\rangle_{\psi_k} \end{aligned} \tag{29}$$

for a projector on the neural basis we get the ensemble average:

$$\left\langle \hat{P}_{\mathbf{s}} \right\rangle = \frac{1}{M} \sum_{k=1}^{M} \left\langle \psi_k \left| \hat{P}_{\mathbf{s}} \right| \psi_k \right\rangle = \frac{1}{M} \sum_{k=1}^{M} \left| \psi_k(\mathbf{s}) \right|^2 \tag{30}$$

Now, if all the members of the ensemble are in the same neural state $|\psi_k\rangle = |\psi\rangle$ for each $k = 1, \ldots, M$ the whole statistical weight that is placed on the projection coincides exactly with $|\psi(\mathbf{s})|^2$ so that the ensemble average of the projection coincides numerically with the degree to which the system projects over the dimension corresponding to the neural pattern $|\mathbf{s}\rangle$

(the projection norm), that is, there is a numerical coincidence between $\left\| \hat{P}_{\mathbf{s}} |\psi\rangle \right\|^2$ and $\left\langle \hat{P}_{\mathbf{s}} \right\rangle$:

$$\left\langle \hat{P}_{\mathbf{s}} \right\rangle = \frac{1}{M} \sum_{k=1}^{M} \left|\psi(\mathbf{s})\right|^2 = \left|\psi(\mathbf{s})\right|^2 \quad (31)$$

Thus, an ensemble of QuANNs with the same structure, characterized by the same quantum state $|\psi\rangle$, has a statistical weight for each projection coincident with the norm of the projection, so that this norm has a statistical expression once we consider an ensemble of systems with the same structure and characterized by the same state.

This is similar to the argument that is made around repeated independent[8] and identically prepared experiments leading to a statistical distribution that shows the markers of the underlying quantum dynamics, in that case, we also see a statistical ensemble marker (considering an ensemble of experiments with the same state vector) that recovers the projection norm structure in the statistical distribution.

The experiments, in the case of human systems, have led to the finding of the same computational properties and projective dynamics present in the quantum systems (Busemeyer and Franco, 2010; Busemeyer and Bruza, 2012; Wang and Busemeyer, 2013; Busemeyer and Wang, 2014), a finding that comes from the statistical distribution of the experiments.

In an econophysics setting, the projective dynamics can be addressed as a cognitive projection such that the projection norm corresponds to the decision weight placed on that alternative[9]. The QuANN state transition has an implication in the projection weights, in the sense that given the state transition:

$$\left|\psi_{out}\right\rangle = \hat{L}_{Net} \left|\psi_{in}\right\rangle = \sum_{\mathbf{s} \in A_2^N} \psi_{out}(\mathbf{s}) \left|\mathbf{s}\right\rangle, \quad (32)$$

the output amplitudes are given by:

$$\begin{aligned} \psi_{out}(\mathbf{s}) &= \sum_{\mathbf{s}' \in A_2^N} \left\langle \mathbf{s} \middle| \hat{L}_{Net} \middle| \mathbf{s}' \right\rangle \left\langle \mathbf{s}' \mid \psi_{in} \right\rangle = \\ &= \sum_{\mathbf{s}' \in A_2^N} L_{Net}(\mathbf{s}, \mathbf{s}') \psi_{in}(\mathbf{s}') \end{aligned} \quad (33)$$

with $L_{Net}(\mathbf{s}, \mathbf{s}') = \left\langle \mathbf{s} \middle| \hat{L}_{Net} \middle| \mathbf{s}' \right\rangle$. Eq. (33) means that the following change in the projections' norms takes place:

$$\begin{aligned} &\left\| \hat{P}_{\mathbf{s}} \left|\psi_{in}\right\rangle \right\|^2 = \left|\psi_{in}(\mathbf{s})\right|^2 \rightarrow \\ &\rightarrow \left\| \hat{P}_{\mathbf{s}} \left|\psi_{out}\right\rangle \right\|^2 = \left|\psi_{out}(\mathbf{s})\right|^2 = \left| \sum_{\mathbf{s}' \in A_2^N} L_{Net}(\mathbf{s}, \mathbf{s}') \psi_{in}(\mathbf{s}') \right|^2 \end{aligned} \quad (34)$$

The sum within the square modulus is a source of quantum interference at the projection norm level.

An iterative scheme with the repeated application of the neural network operator $\hat{L}_{Net}$ leads to a sequence of quantum neural states $|\psi(t)\rangle$. Expanding the complex numbers associated to the quantum amplitudes:

$$\left\langle \mathbf{s} \mid \psi(t) \right\rangle = \psi(\mathbf{s}, t) = \sqrt{A(\mathbf{s}, t)} + i\sqrt{B(\mathbf{s}, t)} \quad (35)$$

We can express the dynamical variables $A(\mathbf{s}, t)$ and $B(\mathbf{s}, t)$ in terms of a dynamical nonlinear state transition rule:

$$A(\mathbf{s}, t) = \left[Re\left(\sum_{\mathbf{s}' \in A_2^N} L_{Net}(\mathbf{s}, \mathbf{s}') \psi(\mathbf{s}', t - \Delta t) \right) \right]^2 \quad (36)$$

$$B(\mathbf{s}, t) = \left[Im\left(\sum_{\mathbf{s}' \in A_2^N} L_{Net}(\mathbf{s}, \mathbf{s}') \psi(\mathbf{s}', t - \Delta t) \right) \right]^2 \quad (37)$$

which leads to a 2^{N+1} system of nonlinear equations, from where it follows that the probability associated to a given neural firing configuration, worked from the expected projection (in accordance with the ensemble average), is given by the sum of the two dynamical variables:

$$Prob[\mathbf{s}, t] = A(\mathbf{s}, t) + B(\mathbf{s}, t) \quad (38)$$

[8] In the case of QuANNs this presupposes the non-interaction between the ensemble elements, appealing to a description of a statistical random sample.

[9] In the quantum computational setting, under the Everettian line, the projective structure for QuANNs can be considered as a computational projection such that each Hilbert space dimension, corresponding to a different neural pattern, is computed simultaneously with an associated weight (given by the norm of the projection), having a computational expression in the system's quantum processing and a statistical correspondence in the neural activity pattern of an ensemble of QuANNs with the same structure and in the same state (assuming non-interaction between different ensemble elements). In the case of physical systems, the projective dynamics, interpreted computationally, leads to a physical expression of the system at multiple dimensions of systemic activity, a point which was interpreted by DeWitt (1970) under the notion of many worlds of a same universe, where each world corresponds to an entire configuration of the universe matching a corresponding orthogonal dimension of an appropriate Hilbert space where observers and systems are correlated (entanglement). In the case of applications to human decision-making, the orthogonal dimensions can be assumed to correspond to alternative decision scenarios evaluated by the decision-maker and supporting his/her choice.

This establishes a bridge between Nonlinear Dynamical Systems Theory and quantum processing by QuANNs, with implications for financial modeling. Indeed, while, traditionally, in financial econometrics one can see the distinction between a stochastic process (be it linear or nonlinear) and a deterministic nonlinear dynamical system, in the case of QuANNs applied to financial modeling they synthesize both approaches (stochastic and deterministic nonlinear), since the quantum state transition equations have a corresponding expression in a nonlinear deterministic dynamical system for probability measures assigned to the QuANN's statistical description via the correspondence between the projection norm dynamics and the statistical expectation associated to the projection operator.

The QuANNs application to financial modeling, thus, allows us to address the problem of simulating the resulting system dynamics that comes from a human cognition where interference patterns are found in the probabilistic description of human behavior.

2.3 A QUANTUM MARKET MODEL

Considering the financial case, a quantum regime switching model for the N volatility components plus the market polarization component, introduced in subsection 2.1, can be addressed through a Quantum Neural Automaton (QNA), defined as a one dimensional lattice with a QuANN associated to each lattice site, in this case we assume the lattice to have $N + 1$ sites and to each site k, for $k = 1,2 \ldots, N + 1$, is associated a QuANN with an architecture defined by a digraph with the following structure:

$$\Gamma = \{(n_1, n_2), (n_2, n_3), (n_3, n_1), (n_3, n_2)\} \quad (39)$$

The corresponding Hilbert space for each such neural network $H_{Net}(k)$ is $H_2^{\otimes 3}$ that is $H_{Net}(k) = H_2^{\otimes 3}$, for $k = 1,2 \ldots, N + 1$, with the general basis vector $|s_1 s_2 s_3\rangle$, such that s_1 characterizes the activity pattern of the first neuron ($n_1(k)$), s_2 characterizes the second neuron ($n_2(k)$) and s_3 characterizes the activity pattern of the third neuron ($n_3(k)$).

In what follows, the neuron $n_3(k)$ encodes the market state for the corresponding component, $n_1(k)$ encodes the new market conditions supporting the corresponding component's dynamics and $n_2(k)$ addresses the computation of the synchronization pattern between $n_3(k)$ (the market state for the component) and $n_1(k)$ (the new market conditions).

The QNA Hilbert space $H_{QNA} = \bigotimes_{k=1}^{N+1} H_{Net}(k)$ is the tensor product of $N + 1$ copies of the Hilbert space $H_2^{\otimes 3}$. Assuming this structure for the QNA, we now begin by addressing the local neural dynamics and its financial interpretation.

2.3.1 Local Neural Dynamics

Since the third neuron firing patterns encode the market state of the corresponding component, for the N volatility components, we have the neural network market volatility operator on $H_2^{\otimes 3}$:

$$\hat{O}_V |s_1 s_2 0\rangle = v_0 |s_1 s_2 0\rangle \quad (40)$$

$$\hat{O}_V |s_1 s_2 1\rangle = v_1 |s_1 s_2 1\rangle \quad (41)$$

while for the market polarization component we have the neural network market polarization operator:

$$\hat{O}_P |s_1 s_2 0\rangle = -1 |s_1 s_2 0\rangle \quad (42)$$

$$\hat{O}_P |s_1 s_2 1\rangle = +1 |s_1 s_2 1\rangle \quad (43)$$

Since, as defined previously, $v_0 \leq v_1$, for a volatility neural network, when the third neuron fires we have a high volatility state, and when it does not fire we have a low volatility state. For the market polarization neural network, when the third neuron fires we have a *bullish* market state and when it does not fire we have a *bearish* market state.

Eqs. (40) to (43) show that both operators depend only on the third neuron's firing pattern, which means that, using Dirac's *bra-ket* notation, they can be expanded, respectively, as:

$$\hat{O}_V = v_0 \left(\sum_{s_1, s_2 \in A_2} |s_1 s_2 0\rangle \langle s_1 s_2 0| \right) + +v_1 \left(\sum_{s_1, s_2 \in A_2} |s_1 s_2 1\rangle \langle s_1 s_2 1| \right) \quad (44)$$

$$\hat{O}_P = -\left(\sum_{s_1, s_2 \in A_2} |s_1 s_2 0\rangle \langle s_1 s_2 0| \right) + +\left(\sum_{s_1, s_2 \in A_2} |s_1 s_2 1\rangle \langle s_1 s_2 1| \right) \quad (45)$$

Now, the neural network follows a closed loop starting at the market state neuron ($n_3(k)$) and ending at the market state neuron. The final state transition amplitudes and the underlying financial dynamics will depend upon the intermediate transformations which may change the profile of the corresponding component's state transition structure.

To address the neural dynamics and its relation with the financial market dynamics we need to introduce the neural links operators and follow the loop, starting at $n_3(k)$ and ending at $n_3(k)$. Considering, then, the first neural link $n_3(k) \to n_1(k)$, we introduce the following neural network operator for the neuron $n_1(k)$:

$$\hat{L}_1 = \sum_{s \in A_2} e^{-\frac{i}{\hbar}\Delta t \hat{H}_0} \otimes |s\rangle\langle s| \otimes |0\rangle\langle 0| + \\ + \sum_{s' \in A_2} e^{-\frac{i}{\hbar}\Delta t \hat{H}_1} \otimes |s'\rangle\langle s'| \otimes |1\rangle\langle 1| \quad (46)$$

using Eq. (19) we need to define the angles $\theta(0)$, $\theta(1)$, $\omega(0)$, $\omega(1)$ and the unit vectors $\mathbf{u}(0)$, $\mathbf{u}(1)$, we set, in this case:

$$\frac{\theta(0)\Delta t}{2} = \varphi + \frac{\pi}{2}, \frac{\theta(1)\Delta t}{2} = \varphi \quad (47)$$

$$\frac{\omega(0)\Delta t}{2} = \pi, \frac{\omega(1)\Delta t}{2} = \frac{\pi}{2} \quad (48)$$

$$\mathbf{u}(0) = \mathbf{u}(1) = (1,0,0) \quad (49)$$

leading to the following operator structure:

$$e^{-\frac{i}{\hbar}\Delta t \hat{H}_0} = \sin(\varphi)\hat{1} + i\cos(\varphi)\hat{\sigma}_1 = \\ = \begin{pmatrix} \sin(\varphi) & i\cos(\varphi) \\ i\cos(\varphi) & \sin(\varphi) \end{pmatrix} \quad (50)$$

$$e^{-\frac{i}{\hbar}\Delta t \hat{H}_1} = i\cos(\varphi)\hat{1} + \sin(\varphi)\hat{\sigma}_1 = \\ = \begin{pmatrix} i\cos(\varphi) & \sin(\varphi) \\ \sin(\varphi) & i\cos(\varphi) \end{pmatrix} \quad (51)$$

the action of the operator $\hat{L}_1$ on the basis states is given by:

$$\hat{L}_1 |s_1 s_2 0\rangle = \sin(\varphi)|s_1 s_2 0\rangle + i\cos(\varphi)|1 - s_1 s_2 0\rangle \quad (52)$$

$$\hat{L}_1 |s_1 s_2 1\rangle = i\cos(\varphi)|s_1 s_2 1\rangle + \sin(\varphi)|1 - s_1 s_2 1\rangle \quad (53)$$

The operator $\hat{L}_1$ can be considered in terms of a quantum regime switching model, such that if the market state neuron $n_3(k)$ is not firing, then, $\sin(\varphi)$ is the amplitude associated to the alternative where the neuron $n_1(k)$ does not change state, while $i\cos(\varphi)$ is the amplitude associated to the alternative where the neuron $n_1(k)$ changes state, on the other hand, if the neuron $n_3(k)$ is firing the role of the amplitudes flip: $i\cos(\varphi)$ is associated with the alternative where the neuron $n_1(k)$ does not change state and $\sin(\varphi)$ is the amplitude associated with the alternative where the neuron $n_1(k)$ changes state.

Before considering the financial implications of this dynamics, it is necessary to address the rest of the network, because the final dynamics and its financial implications can only be fully addressed at the end of the cycle. As we will see, the end result will be a quantum computation-based selection process of adaptive rules regarding market expectations and the processing of how financial news may support trading decisions affecting market polarization and market volume.

Proceeding, then, with the neural links, the second neuron to be activated is $n_2(k)$, which, following the network architecture defined in Eq. (39) receives an input from the two neurons $n_1(k)$ and $n_3(k)$, this neuron will play a key role in the selection of adaptive rules regarding the relation between trading profiles and financial news, a point that we will return to when the final neural network state transition is analyzed. Following the quantum circuit framework, the second neuron is transformed conditionally on the states of the two neurons $n_1(k)$ and $n_3(k)$, in accordance with the neural links $n_1(k) \to n_2(k) \leftarrow n_3(k)$, the corresponding neural links operator is given by:

$$\hat{L}_2 = \sum_{s,s' \in A_2} |s\rangle\langle s| \otimes e^{-\frac{i}{\hbar}\Delta t \hat{H}_{ss'}} \otimes |s'\rangle\langle s'| \quad (54)$$

When the input neurons have synchronized firing patterns, the rotation and phase transformation angles are set to:

$$\frac{\theta(00)\Delta t}{2} = \frac{\theta(11)\Delta t}{2} = 0 \quad (55)$$

$$\frac{\omega(00)\Delta t}{2} = \frac{\omega(11)\Delta t}{2} = 0 \quad (56)$$

which means that the operators reduce to:

$$e^{-\frac{i}{\hbar}\Delta t \hat{H}_{00}} = e^{-\frac{i}{\hbar}\Delta t \hat{H}_{11}} = \hat{1} \quad (57)$$

that is, the second neuron remains in the same state when the input neurons ($n_1(k)$ and $n_3(k)$) exhibit a synchronized firing pattern (no rotation nor phase transformation takes place). When the input neurons do not exhibit a synchronized firing pattern, the rotation and phase transformation is set by the following parameters:

$$\frac{\theta(01)\Delta t}{2} = \frac{\theta(10)\Delta t}{2} = \frac{\pi}{2} \quad (58)$$

$$\frac{\omega(01)\Delta t}{2} = \frac{\omega(10)\Delta t}{2} = \frac{\pi}{2} \quad (59)$$

$$\mathbf{u}(01) = \mathbf{u}(10) = (1,0,0) \quad (60)$$

which leads to:

$$e^{-\frac{i}{\hbar}\Delta t \hat{H}_{01}} = e^{-\frac{i}{\hbar}\Delta t \hat{H}_{10}} = \hat{\sigma}_1 \quad (61)$$

thus, the action of $\hat{L}_2$ on each basis state is such that:

$$\hat{L}_2 |s s_2 s\rangle = |s s_2 s\rangle \quad (62)$$

$$\hat{L}_2 |s s_2 1-s\rangle = |s 1-s_2 1-s\rangle \quad (63)$$

that is, the neuron $n_2(k)$ does not change state when the two neurons $n_1(k)$ and $n_3(k)$ have the same firing pattern, and flips state when the two neurons have differing firing patterns (this is equivalent to a controlled negation quantum circuit).

Now, to close the cycle, and before addressing the final dynamics and its financial interpretation, we have to address, first, the third link $n_2(k) \to n_3(k)$. In this case, we also introduce a controlled-negation circuit, so that the corresponding operator is:

$$\hat{L}_3 = \sum_{s \in A_2} |s\rangle\langle s| \otimes |0\rangle\langle 0| \otimes e^{-\frac{i}{\hbar}\Delta t \hat{H}_0} + \sum_{s' \in A_2} |s'\rangle\langle s'| \otimes |1\rangle\langle 1| \otimes e^{-\frac{i}{\hbar}\Delta t \hat{H}_1} \quad (64)$$

$$\frac{\theta(0)\Delta t}{2} = 0, \frac{\theta(1)\Delta t}{2} = \frac{\pi}{2} \quad (65)$$

$$\frac{\omega(0)\Delta t}{2} = 0, \frac{\omega(1)\Delta t}{2} = \frac{\pi}{2} \quad (66)$$

$$\mathbf{u}(1) = (1,0,0) \quad (67)$$

leading to:

$$e^{-\frac{i}{\hbar}\Delta t \hat{H}_0} = \hat{1}, e^{-\frac{i}{\hbar}\Delta t \hat{H}_1} = \hat{\sigma}_1 \quad (68)$$

so that the basis states transform as:

$$\hat{L}_3 |s_1 0 s_3\rangle = |s_1 0 s_3\rangle \quad (69)$$

$$\hat{L}_3 |s_1 1 s_3\rangle = |s_1 1 1-s_3\rangle \quad (70)$$

these equations show that the neuron $n_3(k)$ changes state when the second neuron is firing and does not change state when the second neuron is not firing. The neural network operator $\hat{L}_{Net}$ is the product of the three operators, that is:

$$\hat{L}_{Net} = \hat{L}_3 \hat{L}_2 \hat{L}_1 \quad (71)$$

Table 1 (in appendix) shows the results of the action of the neural network operator on each basis state.

From a financial perspective, table 1 synthesizes two adaptive rules, one in which the new market state for the component follows the new market conditions underlying the corresponding component's dynamics (neurons' $n_1(k)$ and $n_3(k)$ show a neural reinforcement dynamics), and another in which the new market state is contrarian with respect to the new market conditions underlying the corresponding component's dynamics (neurons' $n_1(k)$ and $n_3(k)$ show a neural inhibitory dynamics). These are two basic rules regarding expectation formation from new data: the decision to follow the new data or not.

In the first case, and taking as example a volatility component, the market is driven by an expectation of continuance of market conditions, so that, for instance, if market conditions are favorable to a high volatility state (neuron $n_1(k)$ is firing), then, the new market state follows the market conditions and $n_3(k)$ fires, corresponding to high volatility.

On the other hand, still under the first adaptive rule, if market conditions are unfavorable to a high volatility state (neuron $n_1(k)$ is not firing), then, the new market state follows the market conditions and $n_3(k)$ does not fire, corresponding to low volatility.

The resulting adaptive rule corresponds, thus, to a *follow the news* rule. Likewise, if we consider, instead, the market polarization component, the *follow the news* rule means that if the new market conditions support a bullish market sentiment, then, the market becomes bullish and if the new market conditions support a bearish market sentiment, then, the market becomes bearish.

The second adaptive rule is the reverse, expectations are that the new market conditions will not hold, and the market does the opposite from the news, expecting speculative gains.

The first adaptive rule is implemented when the second neuron is not firing, while the second rule is implemented when the second neuron is firing. Thus, the firing of the second neuron is a dynamical component that simulates a market change in its expectation and trading profile, so that, for the neural configurations $\{|000\rangle, |001\rangle, |100\rangle, |101\rangle\}$, the state transition for the market component's dynamics is driven by the first adaptive rule, while, for the neural configurations $\{|010\rangle, |011\rangle, |110\rangle, |111\rangle\}$, the market component's dynamics is driven by the second adaptive rule.

While neuron $n_2(k)$'s firing pattern determines the selection of a *follow the news rule*, the combination of firing patterns of the three neurons determines the quantum amplitudes for the market state transitions. Thus, when the neuron $n_2(k)$ is not firing, if the initial market conditions are aligned with the initial market state, then: $\sin(\varphi)$ is the amplitude associated with the alternative in which $n_1(k)$ and $n_3(k)$ transition to a not firing state and $i\cos(\varphi)$ is the amplitude associated with the alternative in which $n_1(k)$ and $n_3(k)$ transition to a firing state.

For a market volatility component, this means that a transition to a high volatility state, supported by market conditions, has an associated quantum amplitude of $i\cos(\varphi)$, while a market transition to a low volatility, state supported by market conditions, has an associated amplitude of $\sin(\varphi)$. The role of these amplitudes switches when $n_1(k)$ and $n_3(k)$ are not initially aligned.

When the neuron $n_2(k)$ is firing, the transition amplitudes to firing/non-firing states follow the same pattern as above for neuron $n_1(k)$ but reverse the pattern for neuron $n_3(k)$ because the new market conditions' neuron and the market state neuron transition to a non-aligned state (the market is contrarian with respect to the news), so that, if $n_1(k)$ and $n_3(k)$ are initially aligned, $\sin(\varphi)$ is the amplitude associated with a transition to the state where $n_1(k)$ is not firing and $n_3(k)$ is firing, while, if $n_1(k)$ and $n_3(k)$ are not initially aligned, the amplitude associated with such a transition is $i\cos(\varphi)$. The roles of the amplitudes, thus, depend upon the way in which the market adapts to new information and the previous configuration of market conditions and market state.

As expected, the market conditions and the market state neurons are always entangled, which means that, in each case, the market state effectively becomes like a measurement apparatus of the market conditions, the entanglement profile can, however, be aligned (*follow the news rule*, based on an expectation of sustainability of the new market conditions) or non-aligned (*contrarian rule*, based on the expectation of reversal of the new market conditions).

Thus, in the model, the quantum neural dynamics models a market that processes the information on the market conditions implementing a standard quantum measurement, but the profile of that quantum measurement depends upon the expectations regarding the news (leading to different entanglement profiles).

The final dynamics for the market component results from the iterative application of the operator $\hat{L}_{Net}$ for each trading round, leading to state transitions between the adaptive rules and, thus, between the market states. Considering a sequence of neural states for the market component's associated neural network $\left|\psi(k,t)\right\rangle$, the state transition resulting from the dynamical rule is given by:

$$\left|\psi(k,t)\right\rangle = \hat{L}_{Net}\left|\psi(k,t-\Delta t)\right\rangle \quad (72)$$

which leads to the following update rule for the quantum amplitudes ψ_k (as per the general Eq. (33)):

$$\begin{aligned}\psi_k(\mathbf{s},t) &= \sum_{\mathbf{s}'\in A_2^3}\left\langle \mathbf{s}\left|\hat{L}_{Net}\right|\mathbf{s}'\right\rangle\left\langle \mathbf{s}' \mid \psi(k,t-\Delta t)\right\rangle = \\ &= \sum_{\mathbf{s}'\in A_2^3} L_{Net}(\mathbf{s},\mathbf{s}')\psi_k(\mathbf{s}',t-\Delta t)\end{aligned} \quad (73)$$

using Table 1's results, in conjunction with this last equation, we obtain the transition table for the quantum amplitudes shown in table 2, provided in the appendix.

Taking into account this general neural dynamics for each component we can now piece it all together to address the market state and resulting financial dynamics.

2.3.2 Financial Market Dynamics

To address the full market dynamics we need to recover the QNA. For each trading round, the quantum state associated with the market dynamics is given by the QNA state defined as the tensor product of the lattice sites' neural networks' states, that is, by the tensor product of each component's neural network state:

$$\begin{aligned}\left|\psi(t)\right\rangle &= \bigotimes_{k=1}^{N+1}\left|\psi(k,t)\right\rangle = \\ &= \Psi(\mathbf{s}_1,\mathbf{s}_2,\ldots,\mathbf{s}_{N+1},t)\left|\mathbf{s}_1,\mathbf{s}_2,\ldots,\mathbf{s}_{N+1}\right\rangle\end{aligned} \quad (74)$$

where the quantum amplitudes $\Psi(s_1, s_2, \ldots, s_{N+1}, t)$ are given by:

$$\Psi(\mathbf{s}_1,\mathbf{s}_2,\ldots,\mathbf{s}_{N+1},t) = \prod_{k=1}^{N+1}\psi_k(\mathbf{s}_k,t) \quad (75)$$

with ψ_k being the amplitudes associated with the lattice site k's neural network.

For the N volatility components we can introduce a corresponding volatility operator on the QNA Hilbert space:

$$\hat{O}_k = \hat{1}^{\otimes k-1} \otimes \hat{O}_V \otimes \hat{1}^{\otimes N+1-k} \quad (76)$$

with $k = 1, \ldots, N$, where, as before, $\hat{1}^{\otimes m}$ denotes m-tensor product of the unit operator on H_2 and $\hat{O}_V$ is the volatility operator defined in Eqs. (40) and (41). Similarly, for the market polarization operator, we write:

$$\hat{O}_{N+1} = \hat{1}^{\otimes N} \otimes \hat{O}_P \quad (77)$$

where $\hat{O}_P$ is the market polarization operator defined in Eqs. (42) and (43). In this way, the returns' dynamical variable defined in Eq. (8) is replaced, in the quantum econophysics setting, by a quantum operator on the QNA Hilbert space defined as:

$$\hat{R} = \frac{1}{\lambda}\prod_{k=1}^{N+1}\hat{O}_k \quad (78)$$

For each basis state of the QNA Hilbert space, the returns operator has an eigenvalue given by the corresponding financial market returns:

$$\hat{R}\left|\mathbf{s}_1,\mathbf{s}_2,\ldots,\mathbf{s}_{N+1}\right\rangle = = R(\mathbf{s}_1,\mathbf{s}_2,\ldots,\mathbf{s}_{N+1})\left|\mathbf{s}_1,\mathbf{s}_2,\ldots,\mathbf{s}_{N+1}\right\rangle \quad (79)$$

with the eigenvalues $R(\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_{N+1})$ given by:

$$R(\mathbf{s}_1,\mathbf{s}_2,\ldots,\mathbf{s}_{N+1}) = \frac{1}{\lambda}\prod_{k=1}^{N} v_k(\mathbf{s}_k)\cdot\sigma_{N+1}(\mathbf{s}_{N+1}) \quad (80)$$

where $v_k(\mathbf{s}_k) = v_0$ if the binary string $\mathbf{s}_k \in A_2^3$ ends in 0 ($n_3(k)$ is not firing) and $v_k(\mathbf{s}_k) = v_1$ if the binary string $\mathbf{s}_k$ ends in 1 ($n_3(k)$ is firing), similarly $\sigma_{N+1}(\mathbf{s}_{N+1}) = -1$ if $\mathbf{s}_{N+1} \in A_2^3$ ends in 0 ($n_3(N+1)$ is not firing) and $(\sigma_{N+1}(\mathbf{s}_{N+1}) = 1$ if $\mathbf{s}_{N+1}$ ends in 1 ($n_3(N+1)$ is firing).

The dynamical rule that comes from the neural networks' quantum computation leads to the market state transition for each trading round:

$$\left|\psi(t)\right\rangle = \bigotimes_{k=1}^{N+1}\hat{L}_{Net}\left|\psi(k,t-\Delta t)\right\rangle \quad (81)$$

leading to the expected value for the returns:

$$\left\langle \hat{R}\right\rangle_t = = \sum_{\mathbf{s}_1,\mathbf{s}_2,\ldots,\mathbf{s}_{N+1}} R(\mathbf{s}_1,\mathbf{s}_2,\ldots,\mathbf{s}_{N+1})\left|\Psi(\mathbf{s}_1,\mathbf{s}_2,\ldots,\mathbf{s}_{N+1},t)\right|^2 \quad (82)$$

so that the market tends to the alternative $R(\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_{N+1})$ with an associated probability of $\left|\Psi(\mathbf{s}_1,\mathbf{s}_2,\ldots,\mathbf{s}_{N+1},t)\right|^2$.

The following figure shows a market simulation on Python 3.4. In the simulations, the initial state for each component is taken from a randomly chosen $U(2)$ gate applied to each neuron with uniform probability over $U(2)$. The figure shows the markers of financial turbulence in the returns, including volatility bursts and jumps.

The main parameters that determine the market profile with regards to turbulence is v_0 and the number of components, the turbulence profile does not change much with respect to the rotation angle φ. Indeed, as shown in the table 3, provided in the appendix, the estimated kurtosis[10] for different simulations with 20 components tends to decrease as v_0 rises. For $v_0 = 0.9$ we no longer find excess kurtosis, the turbulence markers being lost. This approach to low turbulence is progressive as v_0 is raised from 0.8 to 0.9, such that that the price jumps tend to become less severe and less frequent, and the volatility bursts tend to disappear, as shown in Figure 2, in which $v_0 = 0.9$ with the rest of the parameters used in Figure 1's simulation being left unchanged.

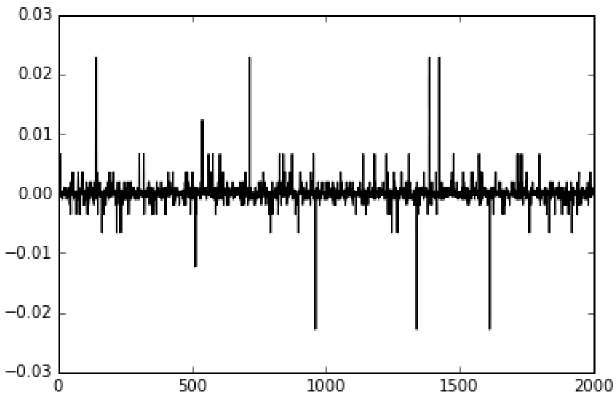

Figure 1. Simulation of the financial returns for $\sin^2\varphi = 0.6$, $v_0 = 0.7$, $\lambda = 1000$, 20 components (19 volatility components plus 1 polarization component). The figure shows 2000 data points of a 2100 data points simulation with the first 100 points removed for transients.

The model, thus, captures different market profiles: as the parameter v_0 increases from 0.8 to 0.9 the simulations tend to approach a lower tail risk dynamics, with a greater approximation to the classical Gaussian returns' probability model occurring for v_0 near 0.87, the table 4 in appendix shows this approximation with the kurtosis values and Jarque-Bera test for normality, as the value of v_0 is increased.

As shown in table 4, for every value of v_0 the Jarque-Bera's null hypothesis is rejected at 1% significance level except for $v_0 = 0.87$. It is important to stress however, that although simulated returns distribution can approximate the Gaussian distribution, this approximation is not robust, different simulations for the same parameters may show deviations from the Gaussian distribution.

Table 5, in appendix, shows examples of simulations for different values of the rotation angle φ, with $v_0 = 0.87$, the null hypothesis of Jarque-Bera's test is not reject, at a 1% significance, for $\sin^2\varphi = 0.1$, 0.3, 0.6, with $\sin^2\varphi = 0.1$, 0.3 as the only cases in which it is not rejected for 5% significance, and $\sin^2\varphi = 0.3$ as

[10] The Fisher kurtosis is used in the statistical analysis of the model's outputs.

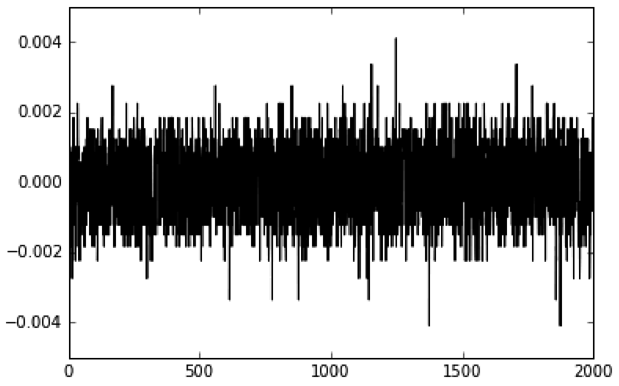

Figure 2. Simulation of financial returns for $\sin^2\varphi = 0.6$, $v_0 = 0.9$, $\lambda = 1000$, 20 components (19 volatility components plus 1 polarization component). The figure shows 2000 data points of a 2100 data points simulation with the first 100 points removed for transients.

the only case in which it is not rejected also for a 10% significance.

These results may, however, depend, as stated previously, upon the simulation, other simulations may show the null hypothesis being rejected for the same parameters, which means that the Gaussian distribution depends upon the sample path and is not a dynamically fixed probability law that can be assumed to hold indefinitely.

The general tail risk pattern, on the other hand, is more robust than the Gaussian approximation, in the sense that as v_0 approaches 0.9 and for $v_0 \geq 0.9$, the market loses the turbulence profile with the jumps and volatility changes becoming less frequent and the kurtosis becoming less and less leptokurtic, leading to lower tail risk, the market returns eventually fluctuate randomly around a narrow band.

Underlying the complex behavior of the simulated market returns is the probability dynamics that comes from the neural network's iterative scheme shown in table 2. Considering Eqs. (35) to (37) and combining with table 2's results we get, in this case, sixteen nonlinear dynamical equations of the general form:

$$A_k(\mathbf{s},t) = \left[\begin{array}{l}\sqrt{A_k(\mathbf{s}',t-\Delta t)}\sin(\varphi) - \\ \sqrt{B_k(\mathbf{s}'',t-\Delta t)}\cos(\varphi)\end{array}\right]^2 \quad (83)$$

$$B_k(\mathbf{s},t) = \left[\begin{array}{l}\sqrt{B_k(\mathbf{s}',t-\Delta t)}\sin(\varphi) + \\ \sqrt{A_k(\mathbf{s}'',t-\Delta t)}\cos(\varphi)\end{array}\right]^2 \quad (84)$$

with $\mathbf{s},\mathbf{s}',\mathbf{s}'' \in \mathrm{A}_2^3$ and $\mathbf{s}' \neq \mathbf{s}''$, so that the probability dynamics that come from the neural network's evolution can be addressed by a nonlinear map with sixteen dynamical variables satisfying the normalization rule:

$$\sum_{\mathbf{s}} A_k(\mathbf{s},t) + \sum_{\mathbf{s}'} B_k(\mathbf{s}',t) = 1 \quad (85)$$

with the probability of the neural configuration **s** being given by the sum:

$$Prob_k[\mathbf{s},t] = A_k(\mathbf{s},t) + B_k(\mathbf{s},t) \quad (86)$$

so that the probability distribution for the neural configurations is a function of a sixteen dimensional nonlinear map on a hypersphere of unit radius (due to the normalization condition).

If we expand the squares in Eqs. (83) and (84) we get:

$$\begin{aligned} A_k(\mathbf{s},t) &= A_k(\mathbf{s}',t-\Delta t)\sin^2(\varphi) + \\ &+ B_k(\mathbf{s}'',t-\Delta t)\cos^2(\varphi) - \\ &- \sqrt{A_k(\mathbf{s}',t-\Delta t)B_k(\mathbf{s}'',t-\Delta t)}\sin(2\varphi) \end{aligned} \quad (87)$$

$$\begin{aligned} B_k(\mathbf{s},t) &= B_k(\mathbf{s}',t-\Delta t)\sin^2(\varphi) + \\ &+ A_k(\mathbf{s}'',t-\Delta t)\cos^2(\varphi) + \\ &+ \sqrt{B_k(\mathbf{s}',t-\Delta t)A_k(\mathbf{s}'',t-\Delta t)}\sin(2\varphi) \end{aligned} \quad (88)$$

which leads to the following expansion for the probability:

$$\begin{aligned} Prob_k[\mathbf{s},t] &= Prob_k[\mathbf{s}',t-\Delta t]\sin^2(\varphi) + \\ &+ Prob_k[\mathbf{s}'',t-\Delta t]\cos^2(\varphi) + \\ &+ \sqrt{B_k(\mathbf{s}',t-\Delta t)A_k(\mathbf{s}'',t-\Delta t)}\sin(2\varphi) - \\ &- \sqrt{A_k(\mathbf{s}',t-\Delta t)B_k(\mathbf{s}'',t-\Delta t)}\sin(2\varphi) \end{aligned} \quad (89)$$

the quantum interference terms (that correspond to the square root terms multiplied by $\sin(2\varphi)$ in Eq. (89)) have an expression, at the probability level, that can be approached in terms of a classical nonlinear dynamical system for the probabilities.

In the classical nonlinear dynamics representation, each financial returns component's stochastic dynamics has a probability measure that updates at each trading round with a deterministic nonlinear update rule, this establishes the bridge between the stochastic process and the nonlinear deterministic dynamical systems modeling of financial dynamics: the neural network's quantum dynamics leads to a nonlinear deterministic dynamics in the probabilities.

A question that may be raised regards the transition from the deterministic nonlinear map to a noisy nonlinear map, from the financial perspective this makes sense since external stochastic factors may affect the financial system. A possible solution for this might be to allow the rotation angle φ to change, so that instead of a fixed value of φ we replace it by a random variable $\varphi_k(t)$ in Eqs. (83) and (84) so that we get a stochastic nonlinear dynamical system. The introduction of a random $\varphi_k(t)$ implies that we are no longer dealing with a fixed unitary operator structure for the QuANN but, instead, work with a quantum neural state transition with a random component in the Hamiltonian, that is, the unitary gates of Eqs. (50) and (51) are now stochastic unitary gates:

$$e^{-\frac{i}{\hbar}\Delta t \hat{H}_0(t)} = \begin{pmatrix} \sin(\varphi_k(t)) & i\cos(\varphi_k(t)) \\ i\cos(\varphi_k(t)) & \sin(\varphi_k(t)) \end{pmatrix} \quad (90)$$

$$e^{-\frac{i}{\hbar}\Delta t \hat{H}_1(t)} = \begin{pmatrix} i\cos(\varphi_k(t)) & \sin(\varphi_k(t)) \\ \sin(\varphi_k(t)) & i\cos(\varphi_k(t)) \end{pmatrix} \quad (91)$$

Thus, a stochastic nonlinear map is induced by the quantum noisy gates in the QuANN's state transition rule, coming from a stochastic Hamiltonian. Figure 3, below, shows the simulation results for:

$$\varphi_k(t) = \arcsin\left(\frac{1}{\sqrt{1+e^{-2\beta z_k(t)}}}\right) \quad (92)$$

with $z_k(t) \sim N(0,1)$, which leads to:

$$\sin^2(\varphi_k(t)) = \frac{1}{1+e^{-2\beta z_k(t)}} \quad (93)$$

$$\cos^2(\varphi_k(t)) = \frac{e^{-2\beta z_k(t)}}{1+e^{-2\beta z_k(t)}} \quad (94)$$

the logistic function present in Eqs. (93) and (94) is also widely used in classical ANNs for the activation probability and leaves room for expansion of connections to Statistical Mechanics (Müller and Strickland, 1995). If we replace in Eq. (89) we get the nonlinear stochastic equations for the probabilities:

$$\begin{aligned} Prob_k[\mathbf{s},t] &= \frac{Prob_k[\mathbf{s}',t-\Delta t]}{1+e^{-2\beta z_k(t)}} \\ &+ \frac{Prob_k[\mathbf{s}'',t-\Delta t]e^{-2\beta z_k(t)}}{1+e^{-2\beta z_k(t)}} + \\ &+ 2\sqrt{B_k(\mathbf{s}',t-\Delta t)A_k(\mathbf{s}'',t-\Delta t)}\frac{e^{-\beta z_k(t)}}{1+e^{-2\beta z_k(t)}} - \\ &- 2\sqrt{A_k(\mathbf{s}',t-\Delta t)B_k(\mathbf{s}'',t-\Delta t)}\frac{e^{-\beta z_k(t)}}{1+e^{-2\beta z_k(t)}} \end{aligned} \quad (95)$$

Figure 3 shows the occurrence of price jumps and clustering volatility, the turbulence in this case is

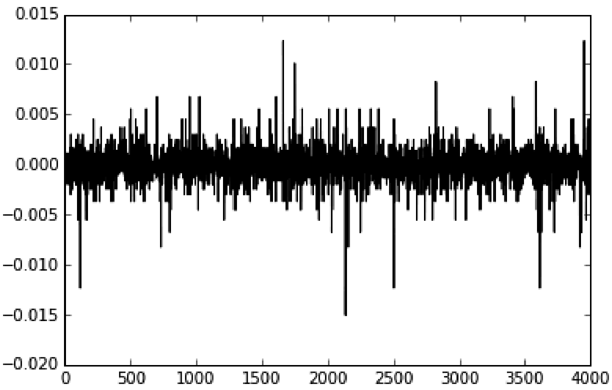

Figure 3. Simulation of financial returns for noisy gates with $\beta = 2.0$, $v_0 = 0.9$, $\lambda = 1000$, 80 components (79 volatility components plus 1 polarization component). The figure shows 4000 data points of a 4100 data points simulation with the first 100 points removed for transients.

linked to the high number of components (rather than to the noisy gates). Indeed, as tables 6. and 7., provided in the appendix, show, the noisy gates do not have a strong effect on the transition from leptokurtic to platikurtic distributions, both for low and high values of β, it is the number of components that has a stronger impact on market profile, as seen in table 7 for the case of $v_0 = 0.88$ which for the simulation with $\beta = 2$ was close enough to the Gaussian distribution for the non-rejection of the null hypothesis of the Jarque-Bera test at a 10% significance level.

Indeed, the number of components shows a strong effect, as can be seen in figure 3, which uses $v_0 = 0.9$ and in table 7, that shows the transition from platikurtic to leptokurtic for large values of the components, for both a low and a high value of β.

3. FINANCE, NONLINEAR STOCHASTIC DYNAMICS AND QUANTUM ARTIFICIAL INTELLIGENCE

From the early onset of development of econophysics, some form of nonlinear stochastic dynamics has been considered to be present in financial market dynamics. A major example being Vaga's work that addressed explicitly different probability distributions corresponding to different (classical) Hamiltonian conditions (Vaga, 1990). The major point that markets make transitions between different regimes and different probability distributions was key to Vaga's market theory. On the other hand, the multifractal multiplicative cascades (Mandelbrot *et al.*, 1997) introduced multiplicative stochastic processes as sources of market turbulence.

While a division line is drawn in regards to nonlinear deterministic processes versus nonlinear stochastic processes, the possible combination of both might provide an intermediate approach, combining adaptive market dynamics and stochastic factors affecting market behavior.

As the previous section model shows, when recurrent QuANNs are applied to financial modeling, the nonlinear deterministic dynamics and the nonlinear stochastic processes result directly from the quantum computational structure, in the sense that: while the iterative computation of a QuANN results from the linear conditional unitary state transition, the corresponding probabilities, due to the square modulus rule for addressing the probabilities associated to different neural firing patterns, leads to a nonlinear update rule for the probabilities themselves, which means that the market behavior will show an interference effect at the probability level expressible in terms of a classical nonlinear map, thus, while the system follows a stochastic dynamics, the probabilities are updated nonlinearly.

This is a direct consequence of quantum cognitive science that comes from human decision analysis, which shows that the nonlinear update in probabilities, leading to non-additive decision weights may be computationally approached from linear unitary quantum computation on an appropriate Hilbert space. Stochastic factors in the nonlinear update of probabilities can also be introduced through unitary noise in the neural network's computation through stochastic Hamiltonians.

Although QuAI and QuANN theory are still on their early stages, they provide a bridge between major lines of research on financial dynamics and risk modeling including: nonlinear deterministic and stochastic dynamics applied to financial modeling, cognitive science and computational foundations of financial theory. Future research on QuANNs dynamics may thus serve as a relevant tool to link different approaches that characterized the different lines of research on econophysics-based finance.

REFERENCES

Anderson, P.W., Arrow, K.J. & Pines, D. (eds.) (1988), *The Economy as an Evolving Complex System*, Perseus Books, USA.

Arthur, B.W., Durlauf, S.N. & Lane, D. (eds.) (1997), *The Economy as an Evolving Complex System II*, Westview Press, USA.

Baaquie, B.E., Kwek, L.C. & Srikant, M. (2000), "Simulation of Stochastic Volatility using Path Integration: Smiles and Frowns", www. arXiv. org.

Baaquie, B.E. & Marakani, S. (2001), "Empirical investigation of a quantum field theory of forward rates", www. arXiv. org.

Baaquie, B.E. (2004), *Quantum Finance: Path Integrals and Hamiltonians for Options and Interest Rates*, Cambridge University Press, USA.

Baaquie, B.E. & Pan, T. (2011), "Simulation of coupon bond European and barrier options in quantum finance", Physica A, 390, 263–289.

Behrman, E.C., Niemel, J., Steck, J.E. & Skinner, S.R. (1996), "A Quantum Dot Neural Network", in Toffoli, T. and Biafore, M. (eds), *Proceedings of the 4th Workshop on Physics of Computation*, Elsevier Science Publishers, Amsterdam, 22–24.

Bohm, D. (1984), *Causality and Chance in Modern Physics*, (1997 Reprinted Version) Routledge & Kegan Paul, Eastbourne.

Bohm, D. & Hiley, B.J. (1993), *The undivided universe — An ontological interpretation of quantum theory*, Routledge, London.

Bransden, B.H. & Joachain, C.J. (2000), *Quantum Mechanics*, Prentice Hall, England.

Bruce, C. (2004), *Schrödinger's Rabbits — the many worlds of quantum*, Joseph Henry Press, Washington DC.

Brunn, C. (ed.) (2006), *Advances in Artificial Economics — The Economy as a Complex Dynamic System*, Springer, Berlin.

Busemeyer, J.R. & Franco, R. (2010), "What is The Evidence for Quantum Like Interference Effects in Human Judgments and Decision Behavior?", NeuroQuantology, vol. 8, No. 4: S48–62.

Busemeyer, J.R. & Bruza, P.D. (2012), *Quantum models of cognition and decision*, Cambridge University Press, Cambridge.

Busemeyer, J.R. & Wang, Z. (2014), "Quantum Cognition: Key Issues and Discussion", Topics in Cognitive Science 6: 43–46.

Calvet, L. & Fisher, A. (2002), "Multifractality in asset returns: Theory and evidence", Review of Economics and Statistics, Vol. 84, 3, August, 381–406.

Calvet, L. & Fisher, A. (2004), "Regime-switching and the estimation of multifractal processes", Journal of Financial Econometrics, 2: 44–83.

Choustova, O. (2007a), "Quantum modeling of nonlinear dynamics of stock prices: Bohmian approach", Theoretical and Mathematical Physics, 152 (2): 1213–1222.

Choustova, O. (2007b), "Toward quantum-like modeling of financial processes", J.Phys.: Conf. Ser. 7001 2006.

Chrisley, R. (1995), "Quantum learning", in Pylkkänen, P. & Pylkkö, P. (eds.), *New directions in cognitive science: Proceedings of the international symposium*, Saariselka, 4–9 August, Lapland, Finland, Finnish Artificial Intelligence Society, Helsinki, 77–89.

Deutsch, D. (1985), "Quantum theory, the Church-Turing Principle and the universal quantum computer", Proc R Soc Lond A, 400–497.

Deutsch, D. (1999), "Quantum theory of probability and decisions", Proc.R.Soc. Lond. A 1999455 3129–3137.

DeWitt, B.S. (1970), "Quantum mechanics and reality — Could the solution to the dilemma of indeterminism be a universe in which all possible outcomes of an experiment actually occur?", Physics Today, Vol. 23, No. 9, 155–165.

Dirac, P.A.M. (1967), *The Principles of Quantum Mechanics (Fourth Edition)*, Claredon Press, Oxford.

Everett, H. (1957), "'Relative state' formulation of quantum mechanics", Rev. of Mod. Physics, 29 (3): 454–462.

Everett, H. (1973), "The Theory of the Universal Wavefunction, PhD Manuscript", in DeWitt, R. & Graham, N. (eds.), *The*

Many-Worlds Interpretation of Quantum Mechanics, Princeton Series in Physics, Princeton University Press, 3–140.

Ehrentreich, N. (2008), *Agent-Based Modeling — The Santa Fe Institute Artificial Stock Market Model Revisited*, Springer, Berlin.

Farmer, D. (2002), "Market force, ecology and evolution", Industrial and Corporate Change, 11 (5): 895–953.

Focardi, S.M. & Fabozzi, F.J. (2004), *The Mathematics of Financial Modeling and Investment Management*, John Wiley and Sons, New Jersey.

Gonçalves, C.P. (2011), "Financial turbulence, business cycles and intrinsic time in an artificial economy", Algorithmic Finance, 1 (2): 141–156.

Gonçalves, C.P. (2013), "Quantum Financial Economics — Risk and Returns", J Syst Sci Complex, 26: 187–200.

Gonçalves, C.P. (2015), "Quantum Cybernetics and Complex Quantum Systems Science — A Quantum Connectionist Exploration", NeuroQuantology, vol. 13, No. 1: 35–48.

Greiner, W, & Müller, B. (2001), *Quantum Mechanics Symmetries*, Springer, Germany.

Haven, E. & Khrennikov, A. (2013), *Quantum Social Science*, Cambridge University Press, New York.

Haken, H. (1977), *Synergetics: An Introduction*, Springer, Germany.

Iori, G. (1999), "Avalanche Dynamics and Trading Friction Effects on Stock Market Returns", Int.J.Mod. Phys. C, 10, 1149.

Ilinski, K. (2001), *Physics of Finance — Gauge Modelling in Non-equilibrium Pricing*, John Wiley and Sons, West Sussex.

Ivancevic, V.G. & Ivancevic T.T. (2010), *Quantum Neural Computation*, Springer, Dordrecht.

Kak, S. (1995), "Quantum Neural Computing", Advances in Imaging and Electron Physics, vol. 94: 259–313.

Kauffman, S.A. (1993), *The Origins of Order — Self-Organization and Selection in Evolution*, Oxford University Press, New York.

Khrennikov, A. (2010), *Ubiquitous Quantum Structure: From Psychology to Finance*, Springer, Berlin.

Khrennikov, A. & Basieva, I. (2014), "Quantum Model for Psychological Measurements: From the Projection Postulate to Interference of Mental Observables Represented As Positive Operator Valued Measures", NeuroQuantology, vol. 12, No. 3: 324–336.

Leggett, A.J. (2002), "Qubits, Cbits, Decoherence, Quantum Measurement and Environment", in Heiss, D. (ed.), *Fundamentals of Quantum Information — Quantum Computation, Communication, Decoherence and All That*, Springer, Germany.

Lux, T. & Marchesi, M. (1999), "Scaling and criticality in a stochastic multi-agent model of a financial market", Nature 397, 498–500.

Lux, T. (2008), "The Markov-Switching Multifractal Model of Asset returns: GMM estimation and linear forecasting of volatility", Journal of Business and Economic Statistics, 26: 194–210.

Mandelbrot, B.B., Fisher, A. & Calvet, L. (1997), "A Multifractal Model of Asset Returns", Cowles Foundation Discussion Papers: 1164.

Mandelbrot, B.B. (1997), *Fractals and Scaling in Finance*, Springer, USA.

McCulloch, W. & Pitts, W. (1943), "A logical calculus of the ideas immanent in nervous activity", Bulletin of Mathematical Biophysics, 7: 115–133.

Menneer, T. & Narayanan, A. (1995), "Quantum-inspired Neural Networks", technical report R329, Department of Computer Science, University of Exeter, Exeter, United Kingdom.

Menneer T. (1998), *Quantum Artificial Neural Networks*, Ph. D. thesis, The University of Exeter, UK.

Müller, B., Reinhardt, J. & Strickland, M.T. (1995), *Neural Networks An Introduction*, Springer-Verlag, Berlin.

Nash, J. (1951), "Non-Cooperative Games", The Annals of Mathematics, Second Series, Vol. 54, No. 2, 286–295.

Nielsen, M. & Chuang, I. (2003), *Quantum Computation and Quantum Information*, The Press Syndicate of the University of Cambridge, UK.

Piotrowski, E.W. & Sładkowski, J. (2001), "Quantum-like approach to financial risk: quantum anthropic principle", Acta Phys. Polon., B32, 3873.

Piotrowski, E.W. & Sładkowski, J. (2002), "Quantum Market Games", Physica A, Vol. 312, 208–216.

Piotrowski, E.W. & Sładkowski, J. (2008), "Quantum auctions: Facts and myths", Physica A, Vol. 387, 15, 3949–3953.

Saptsin, V. & Soloviev, V. (2009), "Relativistic quantum econophysics — new paradigms in complex systems modelling", www. arXiv. org.

Saptsin, V. & Soloviev, V. (2011), "Heisenberg uncertainty principle and economic analogues of basic physical quantities", Computer Modelling and New Technologies, vol. 15, No. 3: 21–26.

Segal, W. & Segal, I.E. (1998), "The Black– Scholes pricing formula in the quantum context", PNAS March 31, vol. 95 no. 7, 4072–4075.

Vaga, T. (1990), "The Coherent Market Hypothesis", Financial Analysts Journal, 46, 6: 36–49.

Voit, J. (2001), *The Statistical Mechanics of Financial Markets*, Springer, New York.

Wallace, D. (2002), "Quantum Probability and Decision Theory, Revisited", www. arXiv. org.

Wallace, D. (2007), "Quantum Probability from Subjective Likelihood: improving on Deutsch's proof of the probability rule", Studies in the History and Philosophy of Modern Physics 38, 311–332.

Wang, Z. & Busemeyer, J.R. (2013), "A quantum question order model supported by empirical tests of an a priori and precise prediction", Topics in Cognitive Science, 5: 689–710.

Zuo, X. (2014), "Numerical Simulation of Asano-Khrennikov-Ohya Quantum-like Decision Making Model", NeuroQuantology, vol. 12, No. 4: 372–385.

APPENDIX – TABLES

Table 1. Neural network operator's action on the basis states.

Basis States	$\hat{L}_{Net}\|\mathbf{s}\rangle = \hat{L}_3\hat{L}_2\hat{L}_1\|\mathbf{s}\rangle$
$\|000\rangle$	$\sin(\varphi)\|000\rangle + i\cos(\varphi)\|111\rangle$
$\|001\rangle$	$i\cos(\varphi)\|010\rangle + \sin(\varphi)\|101\rangle$
$\|010\rangle$	$\sin(\varphi)\|011\rangle + i\cos(\varphi)\|100\rangle$
$\|011\rangle$	$i\cos(\varphi)\|001\rangle + \sin(\varphi)\|110\rangle$
$\|100\rangle$	$i\cos(\varphi)\|000\rangle + \sin(\varphi)\|111\rangle$
$\|101\rangle$	$\sin(\varphi)\|010\rangle + i\cos(\varphi)\|101\rangle$
$\|110\rangle$	$i\cos(\varphi)\|011\rangle + \sin(\varphi)\|100\rangle$
$\|111\rangle$	$\sin(\varphi)\|001\rangle + i\cos(\varphi)\|110\rangle$

Table 2. Update of the quantum amplitudes for a single market component.

New Amplitudes
$\psi_k(000,t) = \sin(\varphi)\psi_k(000,t-\Delta t) + i\cos(\varphi)\psi_k(100,t-\Delta t)$
$\psi_k(001,t) = i\cos(\varphi)\psi_k(011,t-\Delta t) + \sin(\varphi)\psi_k(111,t-\Delta t)$
$\psi_k(010,t) = i\cos(\varphi)\psi_k(001,t-\Delta t) + \sin(\varphi)\psi_k(101,t-\Delta t)$
$\psi_k(011,t) = \sin(\varphi)\psi_k(010,t-\Delta t) + i\cos(\varphi)\psi_k(110,t-\Delta t)$
$\psi_k(100,t) = i\cos(\varphi)\psi_k(010,t-\Delta t) + \sin(\varphi)\psi_k(110,t-\Delta t)$
$\psi_k(101,t) = \sin(\varphi)\psi_k(001,t-\Delta t) + i\cos(\varphi)\psi_k(101,t-\Delta t)$
$\psi_k(110,t) = \sin(\varphi)\psi_k(011,t-\Delta t) + i\cos(\varphi)\psi_k(111,t-\Delta t)$
$\psi_k(111,t) = i\cos(\varphi)\psi_k(000,t-\Delta t) + \sin(\varphi)\psi_k(100,t-\Delta t)$

Table 3. Kurtosis values for different values of $\sin^2\varphi$ and v_0. The other parameters are: $\lambda = 1000$, 20 components (19 volatility components plus 1 polarization component), the Kurtosis coefficient was calculated on 5000 sample data points of a 5100 data points simulation with the first 100 data points removed for transients.

	$\sin^2\varphi = 0.4$	$\sin^2\varphi = 0.5$	$\sin^2\varphi = 0.6$
$v_0 = 0.4$	585.4546	1336.6726	1923.3159
$v_0 = 0.5$	778.0387	1876.3810	783.0852
$v_0 = 0.6$	1015.5296	473.4505	383.9775
$v_0 = 0.7$	77.6054	49.5857	56.8335
$v_0 = 0.8$	6.8827	20.7037	5.6217
$v_0 = 0.9$	−0.8538	−1.2335	−0.9277

Table 4. Kurtosis values and Jarque-Bera test of normality for different values v_0. The other parameters are: $\sin^2\varphi = 0.6$, $\lambda = 1000$, 20 components (19 volatility components plus 1 polarization component), the Kurtosis coefficient was calculated on 5000 sample data points of a 5100 data points simulation with the first 100 data points removed for transients.

	Kurtosis	JB Statistic	p-value
$v_0 = 0.85$	1.2911	353.7757	0.0
$v_0 = 0.86$	0.3596	46.5669	7.7289e-11
$v_0 = 0.87$	0.1746	6.3179	0.0425
$v_0 = 0.88$	−0.0160	76.5797	0.0
$v_0 = 0.89$	−0.6790	106.5589	0.0
$v_0 = 0.9$	−0.8223	143.5983	0.0

Table 5. Kurtosis values for different values φ. The other parameters are: $v_0 = 0.87$, $\lambda = 1000$, 20 components (19 volatility components plus 1 polarization component), the Kurtosis coefficient was calculated on 5000 sample data points of a 5100 data points simulation with the first 100 data points removed for transients.

$\sin^2\varphi$	Kurtosis	JB Statistic	p-value
0.1	0.1285	4.8697	0.0876
0.2	0.6385	84.4538	0.0
0.3	0.0947	3.8794	0.1437
0.4	0.3169	35.1954	2.2773e-08
0.5	0.9213	565.5135	0.0
0.6	0.1746	6.3179	0.0425
0.7	0.0841	29.6076	3.7221e-07
0.8	0.4975	65.859	4.9960e-15
0.9	0.1775	31.3417	1.5640e-07

Table 6. Kurtosis values and Jarque-Bera test of normality p-values for different simulations with varying v_0 and noisy unitary gates. The other parameters are: $\beta = 0.01$ (left table) and $\beta = 2$ (right table), $\lambda = 100$, 20 components (19 volatility components plus 1 polarization component), the kurtosis coefficient was calculated on 5000 sample data points of a 5100 data points simulation with the first 100 data points removed for transients.

$\beta = 0.01$	Kurtosis	JB p-value	$\beta = 2$	Kurtosis	JB p-value
$v_0 = 0.86$	0.9171	0.0	$v_0 = 0.86$	0.7097	0.0
$v_0 = 0.87$	0.0345	1.0518e-07	$v_0 = 0.87$	0.1786	0.0018
$v_0 = 0.88$	−0.3647	2.8422e-13	$v_0 = 0.88$	−0.0856	0.3119
$v_0 = 0.89$	−0.7142	0.0	$v_0 = 0.89$	−0.6205	0.0
$v_0 = 0.9$	−0.6810	0.0	$v_0 = 0.9$	−0.8840	0.0

Table 7. Kurtosis values and Jarque-Bera test of normality for different values of the number of components (N +1) and noisy unitary gates. The other parameters are: $v_0 = 0.88$ $\beta = 0.01$ (left table) and $\beta = 2$ (right table), $\lambda = 100$, the kurtosis coefficient was calculated on 5000 sample data points of a 5100 data points simulation with the first 100 data points removed for transients.

$\beta = 0.01$	Kurtosis	p-value	$\beta = 2$	Kurtosis	p-value
$N + 1 = 10$	−1.3408	0.0	$N + 1 = 10$	−1.3752	0.0
$N + 1 = 20$	−0.3647	2.8422e-13	$N + 1 = 20$	−0.0856	0.3119
$N + 1 = 30$	1.6227	0.0	$N + 1 = 30$	2.2008	0.0
$N + 1 = 40$	2.5130	0.0	$N + 1 = 40$	3.4597	0.0
$N + 1 = 50$	10.1352	0.0	$N + 1 = 50$	15.9802	0.0

Analysis of Long-Term Shareholders Value Drivers: Evidence from UC RUSAL*

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Abstract. The article shows key business value drivers, their importance and applicability in investment decision-making process and in business efficiency analysis. It also shows the correlation between shareholders and stakeholders value. The article presents such approaches of business valuation as market capitalization approach, DCF and EVA approaches of fair value analysis, and fair multiples valuation model. The use of these models shows the main value drivers which enable detailed value creation analysis. The results of models created in this work are tested: we built a real model of business valuation and key value drivers analysis, with evidence from RUSAL Group.

Аннотация. В статье раскрываются ключевые факторы создания стоимости бизнеса, их важность и особенности применения в принятии инвестиционных решений, а также анализе эффективности бизнеса. Раскрывается взаимосвязь между акционерной и стейкхолдерской стоимостью бизнеса. Инструментарий, используемый для анализа, — это методы дисконтированных денежных потоков, добавленной экономической стоимости, а также анализ на базе справедливых мультипликаторов. В результате модификации и применения указанных моделей были проанализированы ключевые драйверы создания стоимости бизнеса. Результаты исследования были протестированы: была построена модель анализа стоимости бизнеса и выявлены ключевые драйверы роста стоимости бизнеса компании «РУСАЛ».

Key words: Business value, EVA, value drivers, business valuation, terminal value, value analysis, financial analysis, ROIC, cash flows, discounted cash flow approach, shareholders value.

1. INTRODUCTION

The value of business is one of the key performance indicators for different economic entities. Many business analysts consider it as the main indicator of company's success especially in long run. If company increases its intrinsic value, it means that it can generate enough cash flows not only to meet its operating needs (cover items of operating expenses) but also to invest in business expansion (cover capital expenditures items).

There are many different performance and efficiency indicators for companies such as revenue, EBITDA, free cash flow, net cash flow, different profitability ratios (return on assets, return on equity, return on invested capital etc.), different liquidity ratios, earnings per share. All these ratios are useful and meaningful in the process of investment decision-making but they also have one disadvantage: not all of them are specific. An investor needs some presumptive figure, which will help him to understand whether to invest in this business. The most appropriate one is the value of analyzed business.

Business valuation can be considered as a part of corporate finance studies. Many corporate finance theories like Modigliani and Miller capital structure theories (capital-structure irrelevance proposition with the assumptions about taxes absence, no transaction costs and no bankruptcy costs, symmetry of market information and similar costs of borrowing for companies and investors), the CAPM (capital assets pricing model) concept, introduced by Jack Treynor, William Sharpe, Lohn Lintner and Jan Mossin, the concept of modern portfolio theory and portfolio diversification, introduced by Harry Markowitz, made a large contribution to the development of business valuation study. These theories were the base for determining risk factors in the process of business valuation. As of determination of free cash flows, connection between earnings and cash flows, the works of R.Brealey and S.Myers such as "Principles of Corporate Finance" contributed a lot in the development of business valuation concepts.

Now we should mention economists who have been developing business valuation as an independent science. One of them is A.Damodaran. In his books about assets and business valuation, he examined the problem of calculating and analyzing business value, basic valuation concepts, different valuation

* Анализ факторов создания долгосрочной акционерной стоимости бизнеса на примере компании «РУСАЛ»

approaches, like discounted cash flow methodology, market approach, assets-based approach. He also analyzed more complicated aspects of business value like different value drivers, special situations (venture capital valuation, early-stage companies' valuation, valuation of companies in the liquidation stages). Problems of valuing business were also analyzed by Jay E.Fishman, Shannon P.Pratt, William J.Morrison. However, in this article we will not analyze general aspects of business valuation; we will try to define key value drivers for business and to build some universal models, which should account the influence of the drivers developed. These problems are in line with the concepts of value-based management. We would mention two most important works dedicated to the business value drivers approach. The first is the economic value added approach, which was introduced by Bennett Steward in his books "The Quest for Value" and "Best-Practice EVA: The Definitive Guide to Measuring and Maximizing Shareholder Value". Another book is written by Tim Koller, the core leader of corporate finance practice at McKinsey & Company, "Valuation: Measuring and Managing the Value of Companies". Both authors pay special attention to the problem of ROIC, WACC, growth and NOPLAT, which are considered key value drivers. This problem will be developed in our article.

The importance of this investigation is not limited by the meaningfulness of business value for different groups of stakeholders but also in approaches used. We will use three approaches for valuation, such as discounted cash flow, discounted economic value added, and fair multiples approach.

2. THEORETICAL ASPECTS OF BUSINESS VALUE. THREE APPROACHES

Fair business value becomes more and more important in life of different societies. In XXI century, possession of useful economic information is one of the success factors. Fair business value is a specific type of information — not available to everyone. That is why it is very important for investors and other groups of stakeholders.. McKinsey & Company made a large research where they found that companies with value-based management are involved in creation of new jobs, increasing GDP and developing scientific progress. Despite the fact that traditionally scientists separate two types of value: shareholders' value and stakeholders' value (we will concentrate on the shareholders' value concept) we can prove that increasing value for shareholders leads to the increased utility for many other stakeholders. In the earlier-mentioned McKinsey & Company's research it was stated that value-oriented companies create healthy business environment and powerful economy, contributing to high standards of living and new opportunities for individuals.

In McKinsey & Company's research different correlations between value growth of large US companies and such factors as employment growth and technological advances were analyzed and found to be positive and strong. It can be proved by Figure 1, which shows this relationship.

If company cuts the costs and uses labor in excess of industry norms and at the same time tries to maximize its value, it will not succeed in the future be-

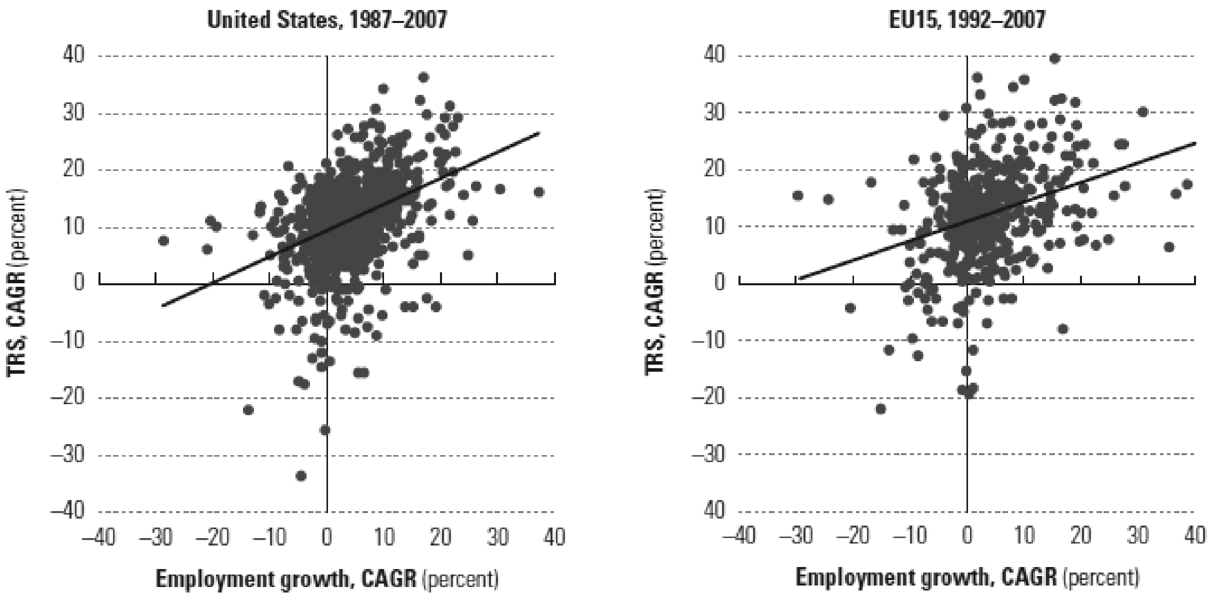

Figure 1. Correlation between value growth and employment growth in USA and Europe.

Source: http://www.mckinsey.com/insights.

cause useful and well-qualified human resources will leave the company and join its peers. This fact will give many competitive advantages for company peers but not for company itself. That is why successful companies offer high salary and bonuses to employees.

Another aspect to be analyzed is correlation between value growth and scientific progress development. McKinsey & Company also analyzed this issue. Results are similar to the previous test: strong positive correlation between the parameters mentioned. Figure 2 could prove this.

Research and development costs are very important for business expansion. They create new products, more effective or less expensive ways of producing goods or rendering services. That is why company oriented on increasing its value is interested in scientific development and progress.

Moreover, companies oriented on value growth in the long-term perspective have a higher level of social and corporate responsibility. They often organize such programs as small enterprises support like VTB or Sberbank in Russia. Value-oriented companies usually have more activities for environment protection. Therefore, we can conclude that companies oriented on the shareholders' value growth also give many benefits for other groups of stakeholders.

We have determined the importance and meaningfulness of increasing shareholders value. Now we will determine formulas and definitions of business value. If we take market capitalization as a beginning point, we can made some corrections and derive the enterprise value formula.

$$EV = Market\,Cap + Debt + Minority + Preffered\ Shares - Cash \quad (1)$$

Where EV — Enterprise value, Market Cap — market capitalization.

Nevertheless, markets are not always efficient and sometimes due to the market speculations, psychology, wrong information they value company in an unfair manner. Such situations are often related to shares' repurchases, mergers and acquisitions and financial engineering. In such cases, analysts need to calculate intrinsic value of business. In this article we will analyze three approaches to determine intrinsic value of business:

1) Discounted cash flow approach;
2) Discounted EVA approach;
3) Fair multiples approach.

2.1 DISCOUNTED CASH FLOW APPROACH

The most commonly used one is discounted cash flow approach. There are two types of cash flows used: free cash flow to equity and free cash flow to firm. The first one means cash flows available for company sharehold-

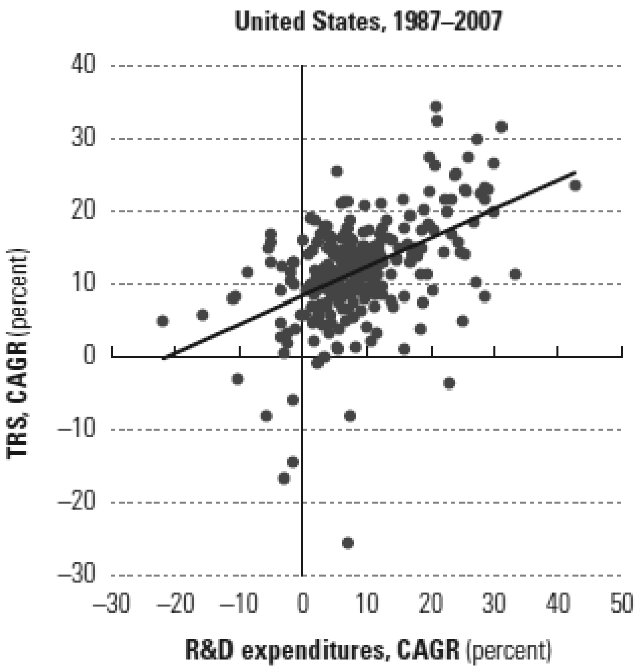

Figure 2. Correlation between value growth and research and development expenditures for US companies.

Source: http://www.mckinsey.com/insights.

ers after all the debt obligations are met, capital expenditures executed and working capital investments met. The second type is broader because it accounts not only shareholders claims but also creditors' claims (more stakeholders involved) and preferred shareholders claims. In our research we will use free cash flow to firm approach (because we don't know the debt and interest schedules for the company analyzed).

Aswath Damodaran in his book "Investment Valuation" defines free cash flow to firm as the sum of all the cash flows to all claim holders in the firm, including stockholders, bondholders and preferred stockholders[1].

There are two ways of calculating free cash flow to firm: from cash flows to equity (formula 2) and from operating income EBIT (formula 3):

$$FCF\left(FCFF\right) = FCFE + IE * \left(1 - T\right) + DPR - ND + D_{ps} \tag{2}$$

Where FCF (FCFF) — free cash flow to firm; IE — interest expenses; T — income tax rate; DPR — debt principal repayment; ND — net debt; D_{ps} — preferred shares dividends.

$$FCF\left(FCFF\right) = EBIT * \left(1 - T\right) + D\,\&\,A - CAPEX - Net\,ch.in WC \tag{3}$$

Where FCF (FCFF) — free cash flow to firm; EBIT — earnings before interest and taxes; T — tax rate; *D&A* — depreciation and amortization; CAPEX — capital expenditures; *Net ch.in WC* — net changes in working capital.

In our modelling process we will use the second formula.

The basic intrinsic value formula is presented as formula 4.

$$Value = \frac{FCF}{WACC - g} \tag{4}$$

Where WACC is weighted average cost of capital and g is organic growth.

We can do some simple mathematic transformations to get the formula which shows the key value drivers (formulas 5–7).

$$FCF = NOPLAT - Net\ Investment = NOPLAT - \left(NOPLAT * IR\right) = NOPLAT * \left(1 - IR\right) \tag{5}$$

$$g = \frac{Net\ Investment}{IC} = \frac{Net\ Investment}{IC} * \frac{NOPLAT}{NOPLAT} = IR * ROIC \tag{6}$$

So we got the model which shows key value drivers:

$$Value = \frac{NOPLAT * (1 - \frac{g}{ROIC})}{WACC - g} \tag{7}$$

But this formula assumes that company's growth rate is a constant in the long run. Commonly companies have two growth periods: unstable growth period (with high or low growth rates) and terminal growth (with steady growth rate). In terminal period company's growth rate is close to the growth rate of GDP in the economy where the company operates (fair value fundamentals concepts). The basic formula for terminal value calculation is presented as formula (8):

$$TV = \frac{FCF_{t+1}}{(WACC - g)^n} \tag{8}$$

We can modify this formula to the model which accounts key value drivers (see formula 9):

[1] A. Damodaran, *Investment Valuation: Tools and Techniques for Determining the Value of Any Asset*, 3rd Edition, p. 533.

$$TV = \frac{\frac{NOPLAT_{t+1} * \left(1 - \frac{g}{ROIC}\right)}{WACC - g}}{\left(1 + WACC\right)^{n}} \qquad (9)$$

Now we can show the complex business model of company value including unstable growth rate period and terminal growth rate period, see formula 10:

$$EV = \sum_{i=1}^{t=n} \frac{FCF_{t}}{\left(1 + WACC\right)^{t}} + \frac{\frac{FCF_{t+1}}{WACC - g}}{\left(1 + WACC\right)^{n}} + Debt + MI + PS - C\&CE \qquad (10)^{2}$$

If we show key value drivers, the formula will be transformed for the next view, see formula 11:

$$EV = \left(\sum_{i=1}^{t=n} \frac{NOPLAT_{t} * \left(1 - \frac{g}{ROIC}\right)}{\left(1 + WACC\right)^{t}} + \frac{\frac{NOPLAT_{t+1} * \left(1 - \frac{g}{ROIC}\right)}{WACC - g}}{\left(1 + WACC\right)^{n}} \right) + b \qquad (11)$$

Where b = $Debt + MI + PS - C\&CE$.

In the formulas above we used the following abbreviations: EV — enterprise value; FCF — free cash flow; WACC — weighted average cost of capital; MI — minority interest; PS — preferred shares; C&CE — cash and cash equivalents.

As we see this model is based on the cash flows from business activity, but cash flows not always can represent the business situation. For example diminishing cash flow can occur in both cases: in poor performance and large capital expenditures. The alternative model is economic value added approach.

2.2 ECONOMIC VALUE ADDED APPROACH

Economic value added was presented by Bennett Steward in his book "The Quest for Value"[3] in 1991. The economic value added shows the value which was created by company over some definite time period over the capital invested. There are three key stones in the definitions of EVA: ROIC (return on invested capital), WACC (the cost of invested capital) and invested capital itself. We can show the formula 12 which presents relations between the exponents above:

$$EVA = IC * (ROIC - WACC) \qquad (12)$$

Where IC is invested capital, ROIC and WACC are defined above.

The key advantage of EVA is that it shows the main value drivers, but it also has some disadvantages, for example financial managers of companies firstly couldn't apply EVA approach for vertical and horizontal comparisons with peers and also there are some difficulties with differences in scale. Some difficulties are also connected with using EVA as a KPI for bonus programs. In investment services the situation is quite different. For qualified investors EVA is one of key indicators of stable company development and future business potential. The first bulge bracket banks which started using EVA for companies' valuation are Goldman Sachs and Credit Suisse. Then Bennett Steward founded his own corporation EVA Dimensions[4] which aimed to develop value-based management techniques for different companies and EVA approaches for business valuation.

[2] A. Damodaran, *Investment Valuation: Tools and Techniques for Determining the Value of Any Asset*, 3rd Edition. — p. 1500.
[3] G. Bennett Stewart, *The Quest for Value: A Guide for Senior Managers*, HarperCollins Publishers, New-York, 1999.
[4] http://www.evadimensions.com.

Business valuation based on the economic value added approach becomes more and more popular and useful. As mentioned above, this method allows determining key value drivers such as ROIC, WACC and organic growth. It as an analogue of discounted cash flow approach and should show the same results but with another specification. In our analysis we can prove it using some not very complicated mathematical transformations. We have already shown the transformation from common DCF approach to the approach with key value drivers, it was shown in formula 7:

$$Value = \frac{NOPLAT * (1 - \frac{g}{ROIC})}{WACC - g} \quad (7)$$

We can restate the formula 7 in the following way:

$$Value = \frac{IC * ROIC * (1 - \frac{g}{ROIC})}{WACC - g} = IC * \frac{ROIC - g}{WACC - g} \quad (13)$$

We will continue our mathematic transformations and add WACC to the numerator and then subtract it back in formula 14:

$$Value = IC * \frac{ROIC - g + WACC - WACC}{WACC - g} = \frac{IC * \left(ROIC - WACC\right)}{WACC - g} + \\ + \frac{IC * \left(WACC - g\right)}{WACC - g} = IC + \frac{IC * (ROIC - WACC)}{WACC - g} \quad (14)$$

We got economic value added formula. It represents business value of the company with stable growth rate as a sum of invested capital and economic value added. Now let's develop the idea and show enterprise value in the two growth stages periods (unstable and stable growth rates). The common view of this formula id following:

$$Value = IC_0 + \sum_{i=1}^{t} \frac{IC_{i-1} * (ROIC_i - WACC)}{(1 + WACC)^i} + \frac{Terminal\,Value}{(1 + WACC)^t} \quad (15)$$

Where IC_0 is invested capital as of the date of valuation, t — projection period.

Terminal value is the present value of future cash flows beyond the projection period[5]. The key factors of terminal value calculation are terminal organic growth and terminal cost of capital (WACC). The analysis of terminal value calculation is presented in the formulas 16–18.

$$TV = EVA_{t+1} + EVA_{t+2} = \frac{IC_t * (ROIC_t - WACC)}{WACC} + \frac{PV(EVA_{t+2})}{WACC - g} \quad (16)$$

In the formula $PV(EVA_{t+2})$ is determined in the following way:

$$PV\left(EVA_{t+2}\right) = \frac{NOPLAT_{t+1} * \left(\frac{g}{ROIC}\right) * (ROIC - WACC)}{WACC} \quad (17)$$

[5] Tim Koller , Richard Dobbs , Bill Huyett, Value: *The Four Cornerstones of Corporate Finance by McKinsey & Company Inc.*, John Wiley & Sons, Inc., Hoboken, New Jersey, 2011, p. 213.

So we analyzed all the aspects of business valuation using economic value-added approach and can combine them in the resulting model:

$$Value = IC_0 + \sum_{i=1}^{t} \frac{IC_{i-1} * \left(ROIC_i - WACC\right)}{\left(1+WACC\right)^i} + \frac{\frac{IC_t * (ROIC_t - WACC)}{WACC} + \frac{\frac{NOPLAT_{t+1} * \left(\frac{g}{ROIC}\right) * (ROIC - WACC)}{WACC}}{WACC - g}}{(1+WACC)^t} \quad (18)$$

2.3 FAIR MULTIPLES APPROACH

Multiple is a ratio which shows relation between equity or enterprise value and parameter. There are different goals for calculating multiples: comparing current company value with its historical figures, with peers' value, or with sector figure. There is one more type of multiples: fair multiples or target multiples. Fair multiple presents maximum price which a knowledgeable investor is ready to pay with stated values of key value drivers and which allows the investor to get fair required rate of return (IRR analogue). Fair multiples help the investor or analyst to determine key value drivers and to analyze company in a proper way.

There are two ways how to calculate fair multiples, the choice depends on the growth stage of company. If company's growth rates are stable, and we have some adequate grounds for assuming that rated to be constant in the foreseeable future, the one-step model should be used. Nevertheless, in practice more common solution is to use two-step model, which divides foreseeable period on two stages: initial unstable growth stage and terminal steady growth stage. The second variant will be used in our analysis (the detailed analysis will be provided on the base of EV/NOPLAT, net operating profit less adjusted taxes multiple). We will start the common value formula from the first part.

$$Value = \frac{NOPLAT * (1 - \frac{g}{ROIC})}{WACC - g} \quad (7)$$

We will transfer it to the following model:

$$EV = \frac{NOPLAT * (ROIC - g)}{ROIC * (WACC - g)} \quad (19)$$

When using target multiples we assume that in the terminal period ROIC equals WACC (the concept of company growth with a zero value added). Based on this equation we will get the following model for terminal period:

$$TV = \frac{NOPLAT}{WACC} \quad (20)$$

Therefore, we got the base for calculating enterprise value. Formula 20 is a cash flow from period n. Using the concept of target multiples we can propose that cash flow as an annuity with n years' time horizon, see formula 21.

$$EV = \frac{NOPLAT * (ROIC - g)}{ROIC * (WACC - g)} * (1 - \frac{\left(1+g\right)^n}{\left(1+WACC\right)^n}) \quad (21)$$

We should also transform NOPLAT for the period n to $NOPLAT_{n+1}$ and then find present value of the terminal value used.

$$TV = \frac{NOPLAT*(1+g)^n}{WACC} * \frac{1}{(1+WACC)^n} \tag{22}$$

Now we will combine the whole model (two-stages):

$$EV = \frac{NOPLAT*(ROIC-g)}{ROIC*(WACC-g)} * \left(1 - \frac{(1+g)^n}{(1+WACC)^n}\right) + \frac{NOPLAT*(1+g)^n}{WACC} * \frac{1}{(1+WACC)^n} \tag{23}$$

For getting target multiple we should divide the whole model by NOPLAT:

$$\frac{EV}{NOPLAT} = \frac{(ROIC-g)}{ROIC*(WACC-g)} * \left(1 - \frac{(1+g)^n}{(1+WACC)^n}\right) + \frac{1}{WACC} * \frac{(1+g)^n}{(1+WACC)^n} \tag{24}$$

The similar logic was used for calculating EV/EBIT, EV/EBITDA, EV/Sales multiples. We will present only results:

$$\frac{EV}{EBIT} = \frac{(ROIC-g)*(1-T)}{ROIC*(WACC-g)} * \left(1 - \frac{(1+g)^n}{(1+WACC)^n}\right) + \frac{1}{WACC} * (1-T) * \frac{(1+g)^n}{(1+WACC)^n} \tag{25}$$

$$\frac{EV}{EBITDA} = \frac{(ROIC-g)*(1-T)(1-D)}{ROIC*(WACC-g)} * \left(1 - \frac{(1+g)^n}{(1+WACC)^n}\right) + \frac{(1-T)*(1-D)}{WACC} * \frac{(1+g)^n}{(1+WACC)^n} \tag{26}$$

$$\frac{EV}{Sales} = \frac{(ROIC-g)*(1-T)*M}{ROIC*(WACC-g)} * \left(1 - \frac{(1+g)^n}{(1+WACC)^n}\right) + \frac{(1-T)*M}{WACC} * \frac{(1+g)^n}{(1+WACC)^n} \tag{27}$$

Formulas above have some limitations when WACC equals growth. In that case we should add infinitesimal change to g, (0.0000001), such change does not affect the results but helps to avoid dividing by zero.

When analyzing business it is very important to choose the right multiple. McKinsey & Company decided that the best multiple for a variety of companies from S&P 500 is EV/EBITA. It is close to EBITDA but it takes into account depreciation expenses as a part of operating expenses necessary for company to maintain its fixed assets. We will provide a model for EV/EBITA target multiple, where A is the rate of depreciation.

Formula 28 presents one stage model:

$$\frac{EV}{EBITA} = \frac{(ROIC-g)*(1-T)(1-A)}{ROIC*(WACC-g)} * \left(1 - \frac{(1+g)^n}{(1+WACC)^n}\right) + \frac{(1-t)*(1-A)}{WACC} * \frac{(1+g)^n}{(1+WACC)^n} \tag{28}$$

Formula 29 presents two stage model:

$$\frac{EV}{EBITA} = \frac{(ROIC-g)*(1-T)(1-A)}{ROIC*(WACC-g)} * \left(1 - \frac{(1+g)^n}{(1+WACC)^n}\right) + \\ + \frac{(ROIC_{LT}-g_{LT})*(1-T)*(1-A)}{ROIC_{LT}*(WACC_{LT}-g_{LT})} * \frac{(1+g)^n}{(1+WACC)^n} \tag{29}$$

We analyzed the basic concepts of business valuation using value drivers approach and created models for three cases: discounted cash flow methodology, discounted economic value added and target multiples concept.

Now we will present the results of model testing evidence from UC RUSAL, the largest aluminum producing company in Russia, one of the world leaders.

3. UC RUSAL VALUATION AND ANALYSIS ON THE BASE OF MODELS DEVELOPED

3.1 DISCOUNTED CASH FLOW METHODOLOGY

The valuation was provided as of 30.06.2014, based on the macroeconomic assumptions as of 30.06.2014. For terminal growth rate, we used 4% GDP growth for Russia, a long-term estimation by Bloomberg.

The first step after determining assumptions (macro and micro ones) is revenue analysis and estimation. Revenue is the key element for estimating other elements of free cash flow. The results of revenue estimation are presented in the Table below:

The revenue was estimated using bottom-up approach. For calculations were used such parameters as historical company's revenue dynamics, aluminum price forecast, contraction in manufacturing data, provided by UC RUSAL, GDP growth estimations and PPI forecast (data provided by Bloomberg).

The next step in analysis is estimation of costs of goods sold. In analysis were used such data as PPI estimation, provided by Bloomberg, UC RUSAL contraction in manufacturing estimates, foreign exchange forecasts for USD/RUB provided by Bloomberg. The results are presented in Table 2.

For estimations the following expense structure was used: inventory — 55%, energy — 26%, salary — 10%, transportation — 6%, other — 3%. Such structure was provided by company management.

The next calculation is operating expenses, which were calculated as a percentage of sales. The results are presented in Table 3.

The next step is to calculate fixed assets, depreciation expenses and capital expenditures (the results are provided in Table 4).

The next step is net working capital estimation. Each of working capital components was estimated in accordance with its driver (revenue or COGS). The results are presented in Table 5.

The next step is free cash flow (to firm) calculation. The results are presented in Table 6.

The next step is weighted average cost of capital calculation, the results presented in Table 7.

In calculations we used beta which was calculated as a median line of peers' betas.

The present value of estimated cash flows using calculated WACC is 2 870.64 ml USD. We should correct this value for current debt level and cash position.

The terminal value of RUSAL will be calculated using Gordon growth model. We assume last year estimated cash flow to be equal to 868 ml USD, terminal WACC — 13.48%, growth rate — 4%.

$$TV = \frac{868}{13,48\% - 4\%} = 9\,160,46 \text{ ml USD.}$$

Present value of the figure received equals 4 868.72 ml USD. Current level of company debt is 10 892 ml USD, cash — 552 ml USD. Enterprise value can be calculated as following:

Table 1. UC RUSAL revenue estimation, ml USD.

Year	2013	2014	2015	2016	2017	2018
1	7	8	9	10	11	12
Revenue	**9760**	**9095**	**9982**	**10836**	**11181**	**11591**
Growth rate	90%	93%	110%	109%	103%	104%
Aluminum Price	1800	1898	1935	2065	2202	2315
Producers price index	87%	105%	102%	107%	107%	105%
Contraction in manufacturing	92%*	92%				
Real GDP dynamics (Russia), % (yoy)	101%	101%	102%	102%	102%	102%
Corrections for demand growth			101%	101%	101%	101%

Source: Calculated by author.

* www.rusal.ru

Table 2. US RUSAL costs of goods sold estimation, ml USD.

Year	2013	2014	2015	2016	2017	2018
1	**7**	**8**	**9**	**10**	**11**	**12**
COGS	**8312**	**6964**	**7477**	**8123**	**8386**	**8693**
Contraction in manufacturing	92%	92%				
Economy reserves	92%	92%				
Inventories	4538	3802	4160	4552	4981	5451
Energy	2184	1830	1992	2238	2616	2862
PPI	104%	110%	109%	109%	109%	109%
USDRUB	33	36	36	35	34	34
Change % yoy	108%	108%	100%	99%	97%	100%
Salary	852	714	750	787	826	866
Inflation	7%	6%	5%	5%	5%	5%
Transportation expenses	497	417	456	499	546	597
Other	240	201	212	222	233	245

Source: Calculated by author.

Table 3. UC RUSAL operating expenses estimation, ml USD.

Year	2013	2014	2015	2016	2017	2018
1	**7**	**8**	**9**	**10**	**11**	**12**
Operating expenses	**–404**	**1 037**	**1 138**	**1 235**	**1 275**	**1 321**
Revenue	9 760	9 095	9 982	10 836	11 181	11 591
OPEX as a % of revenue	–4%					
Median line (2008–2013)	12%					

Source: Calculated by author.

$$EV = 2870{,}64 + 10892 - 552 + 4868{,}72 = 18079{,}36\, ml\, USD$$

Today's UC RUSAL value equals 18–20 ml USD which lies in line with calculated figures.

3.2 DISCOUNTED ECONOMIC VALUE ADDED METHODOLOGY

The first step is to calculate invested capital, which is determined as a sum of net working capital, fixed assets, intangible assets and other operating assets less liabilities. The results are provided in Table 8.

The calculation uses as a percentage of sales estimation methodology.

The next step is EVA calculation. We used long term steady WACC provided by JP Morgan research (Global research 31/07/2014). The results are presented in Table 9.

Negative values of EVA are results of low aluminum prices, in the long-term negative EVA will turn to positive.

After discounting received EVA figures using current WACC we will get a sum of discounted EVA of –2 341.8 ml USD.

The next step is terminal value calculation. The results are provided below in accordance with 16–18 formulas.

$$EVA_{t+1} = \frac{IC_t * (ROIC_t - WACC)}{WACC} = \frac{23404 * (5\% - 7\%)}{7\%} = -3999{,}07 \text{ ml USD.}$$

Table 4. UC RUSAL capital expenditures estimation, ml USD.

Year	2013	2014	2015	2016	2017	2018
1	**7**	**8**	**9**	**10**	**11**	**12**
Fixed assets	**4887**	**4554**	**4998**	**5425**	**5598**	**5804**
As a % of revenue						
Median line	50,1%					
Change	-566	-333	444	427	173	205
Depreciation	**799**	**525**	**577**	**626**	**646**	**670**
As a % of revenue	8,2%					
Median line	5,8%					
Amortization	**13**	**13**	**14**	**15**	**15**	**16**
As a % of sales	0,0%					
Median line	0,0%					
Revenue	9760	9095	9982	10836	11181	11591
Capital expenditures	246	205	1035	1068	834	891

Source: calculated by author.

Table 5. UC RUSAL net working capital estimation, ml USD.

Year	2013	2014	2015	2016	2017	2018
1	**7**	**8**	**9**	**10**	**11**	**12**
Assets						
Cash and cash equivalents	300	300	300	300	300	300
Receivables	177	165	181	196	202	210
Median line (% of sales)	2%					
Inventories	2663	2231	2396	2603	2687	2785
Median line (% of sales)	32%					
Other operating assets	698	650	714	775	800	829
Median line (% of sales)	7%					
Assets	**3661**	**3182**	**3410**	**3677**	**3787**	**3914**
Liabilities						
Payables	838	781	857	931	960	996
Median line (% of COGS)	9%					
Other short-term liabilities	1130	947	1017	1105	1141	1182
Median line (% of COGS)	14%					
Sales	9760	9095	9982	10836	11181	11591
COGS	8312	6964	7477	8123	8386	8693
Liabilities	**1969**	**1728**	**1874**	**2035**	**2101**	**2178**
Net working capital	**1692**	**1453**	**1535**	**1642**	**1686**	**1736**
Change in net working capital	**–188**	**–239**	**82**	**107**	**44**	**51**

Source: calculated by author.

Table 6. UC RUSAL free cash flow calculation, ml USD.

Year	2013	2014	2015	2016	2017	2018
1	7	8	9	10	11	12
Sales	9760	9095	9982	10836	11181	11591
COGS	8312	6964	7477	8123	8386	8693
Gross profit	1448	2131	2506	2713	2795	2898
Operating expenses	−404	1037	1138	1235	1275	1321
EBIT	1852	1094	1368	1478	1521	1577
Depreciation	799	525	577	626	646	670
Amortization	13	13	14	15	15	16
EBITDA	2665	1632	1958	2119	2182	2262
NOPLAT	1482	876	1094	1182	1216	1261
Adjusted depreciation	639	420	461	501	517	536
Adjusted amortization	11	10	11	12	12	13
Change in net working capital	−188	−239	82	107	44	51
Capital expenditures	246	205	1035	1068	834	891
Free cash flow	**2073**	**1340**	**450**	**520**	**868**	**868**

Source: calculated by author.

Table 7. UC RUSAL WACC.

Category	Designation	Value	Source
1	2	3	4
Risk free rate, USD yield	Rf	5,50%	Bloomberg, eurobonds Russia-2028 REGS
Unlevered beta	βunlev	1,07	Bloomberg, peers analysis
Debt/Equity ratio	D/E	1,04	Bloomberg, peers analysis
Weight of debt	Wd	71%	Bloomberg
Weight of equity	We	29%	Calculation
Levered beta	βrelev	1,43	Calculation
Equity risk premium	ERP	6,97%	Calculation
Cost of equity	Ke (USD)	15,46%	Calculation
Eurobonds Russia 2028 yield	YTM USD	5,50%	Bloomberg, eurobonds Russia-2028 REGS
OFZ 2027 yield	YTM RUB	9,12%	Bloomberg, OFZ 2027
Cost of long-term debt	Kd (USD)	11,00%	Bloomberg
Income tax rate	t	20%	Internal Revenue Code
Weighted average cost of capital	**WACC (USD)**	**13,48%**	

Source: calculated by author.

Table 8. UC RUSAL invested capital, ml USD.

Year	2013	2014	2015	2016	2017	2018
1	**7**	**8**	**9**	**10**	**11**	**12**
Net working capital	1692	1453	1535	1642	1686	1736
Net fixed assets	4887	4554	4998	5425	5598	5804
Net intangible assets	3397	3209	3522	3823	3945	4090
Other	9126	9238	10140	11006	11357	11774
Invested capital	**19102**	**18454**	**20195**	**21897**	**22586**	**23404**

Source: calculated by author.

Table 9. UC RUSAL EVA calculation, ml USD.

Year	2013	2014	2015	2016	2017	2018
1	**7**	**8**	**9**	**10**	**11**	**12**
NOPLAT	1482	876	1094	1182	1216	1261
ROIC	8%	5%	5%	5%	5%	5%
WACC	7%	13%	8%	7%	7%	7%
EVA	**202**	**–1612**	**–600**	**–241**	**–252**	**–260**

Source: calculated by author.

In the process of EVA_{t+2} calculation we used the following estimations: long-term growth as 4%, long-term WACC as 6.55%, long-term ROIC as 8.5% as of the date of estimation, provided by JP Morgan.

$$EVA_{t+2} = \frac{PV\left(EVA_{t+2}\right)}{WACC-g} = \frac{\dfrac{NOPLAT_{t+1} * \left(\dfrac{g}{ROIC}\right) * \left(ROIC - WACC\right)}{WACC}}{WACC-g} =$$

$$= \frac{\dfrac{1312 * \left(\dfrac{4\%}{8.55\%}\right) * (8.5\% - 6.55\%)}{6.55\%}}{6.55\% - 4\%} = 7\,206.90\, ml\, USD$$

Enterprise value calculated on the base of EVA approach is calculated in the following way:

$$Value = IC_0 + \sum_{i=1}^{t} \frac{IC_{i-1} * (ROIC_i - WACC)}{(1+WACC)^i} + \frac{Terminal\, Value}{(1+WACC)^t} =$$
$$= 19102{,}08 + \left(-2341{,}8\right) + 1704{,}94 = 18\,465{,}21\, ml\, USD$$

As we can see the figure of calculated value using EVA approach is in line with the value figure from DCF approach.

3.3 TARGET MULTIPLES METHODOLOGY

We will use EV/EBITDA multiple in this approach. The formula used is provided below (from the first part of article):

$$\frac{EV}{EBITDA} = \frac{\left(ROIC-g\right)*(1-T)(1-D)}{ROIC*(WACC-g)} * \left(1 - \frac{\left(1+g\right)^n}{\left(1+WACC\right)^n}\right) + \frac{(1-T)*(1-D)}{WACC} * \frac{\left(1+g\right)^n}{\left(1+WACC\right)^n} \quad (26)$$

We used the following fundamentals determined on the base of historical analysis and analysts reports: ROIC — 6%, amortization rate — 0.4%, depreciation rate — 10%, projected period growth — 6.3%, tax rate — 20%, projected period WACC — 6.6%, long-term WACC — 6.5%, long-term ROIC — 8.5%, long-term growth rate — 4%.

The results are following:

$$\frac{EV}{EBITDA}=\frac{(6\%-6,3\%)*(1-20\%)(1-10\%)(1-0,4\%)}{6\%*(6,5\%-6,3\%)}*\left(1-\frac{(1+6,3\%)^5}{(1+6,5\%)^5}\right)+$$
$$+\frac{(1-20\%)*(1-10\%)(1-0,4\%)}{6,55\%}*\frac{(1+4\%)^5}{(1+6,5\%)^5}=9,74$$

Using the received multiple and 2013 EBITDA of 2 255 ml USD, enterprise value should be equal to 21 972.4 ml USD.

So we can conclude that using target multiples enterprise value should be equal to 22–23 ml USD without any significant crises and changes in global economic conditions.

The difference between value figures calculated using DCF approach, EVA approach and target multiples approach is insignificant. So the proposed models provide similar results close to reality and can be used for business value analysis.

4. FINDINGS AND CONCLUSION

In the article we presented new value drivers oriented valuation methodologies on the base of DCF, EVA and target multiples approaches. We proved that results from all three approaches are similar and can be used in practice. We created the universal approach which determines business value in terms of mixed influence of such value drivers as ROIC, WACC, NOPLAT, organic growth. This approached is proved in all three models presented. Moreover we can create a wider base of value drivers by decomposing and analyzing each of above-mentioned ones. For example ROIC can be decomposed using DuPont-analogue model in the following way: $ROIC=\frac{NOPLAT}{IC}=(1-T)*\frac{EBIT}{Sales}*\frac{Sales}{IC}=ROS*Capital\,Turnover*(1-T)$. Such types of analysis can be applied to other value factors.

So it shows that the proposed methodology presents traditional approaches such as DCF and created a scheme of developing them in terms of more accurate future value forecasting for meeting needs of different stakeholders classes.

Today the most common valuation practice is ordinary DCF analysis. Our approach enriches traditional DCF, proposes practical implementation of using EVA and target multiples approaches. The models can also be used in trading strategies algorithms: buying shares when the observed multiple below the target one and selling when the observed is above the target one.

The models were tested on the base of UC RUSAL, one of the world's largest aluminum producers. As of the beginning of estimation the company was valued by Bloomberg at 12 000 ml USD, 1.5 years after the company valuation was increased by 60–80% to near 20 000 ml USD.

The models created help to combine external investment analysis with detailed value drivers' analysis. Models help to determine the direction of future value dynamics, take effective investment decisions for direct and portfolio investments. The model recommendations were used for investment decisions by Gazprombank Asset Management and resulted in good profit figures.

REFERENCES

Damodaran A. (2012), *Investment Valuation: Tools and Techniques for Determining the Value of Any Asset, 3rd Edition,* New York: Wiley.

Stewart G.Bennett (1999), *The Quest for Value: A Guide for Senior Managers*, New-York: HarperCollins Publishers.

Koller T., Goedhart M., Wessels D. (2010), *Valuation Measuring and Managing the Value of Companies*, New Jersey: JohnWiley & Sons, Inc., 4.

Modigliani, F. and Miller, M.H. (1958). The Cost of Capital, Corporate Finance and the Theory of Investment, *American Economic Review*, 48, 261–97.

Marshall A., (1890) *Principles of Economics, vol. 1,* New York: Macmillan, 142.

Freeman R.Edward (1984) *Strategic Management: A stakeholder approach*.— Boston: Pitman.

Pratt, Shannon P. (2002), *Cost of capital: estimation and applications*: New York: John Wiley & Sons, Inc., p. 5

S.A.Ross (1976) "The Arbitrage Theory of Capital Asset Pricing", *Journal of Economic Theory*, p. 341–60

Edward S.Mason (1939), „Price and Production Policies of Large-Scale Enterprise", *American Economic Review*, (1, Supplement), pp. 61–74.

Michael E.Porter (1980), *Competitive strategy*, New York: The Free Press.

Koller T., Dobbs R, Huyett B (2011), *Value: The Four Cornerstones of Corporate Finance by McKinsey & Company Inc.*, New Jersey: John Wiley & Sons, p. 41.

Success Determinants of Crowdfunding Projects*

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Abstract. Estimation of social network's *willingness to help* is a key factor in decision making, when setting up a crowdfunding (CF) campaign. This study seeks to identify the effect of past experience on social engagement attitude, such as commitment to participate in crowdfunding activities. We explore impact of differences in investor's utilities from participating in crowdfunding, related to investor's beliefs, behavioral patterns and background such as entrepreneurial experience and motivation to attempt (assumed or factual), attitude to gambling, career preferences and some other. In addition to self-reported survey data (N = 120), we analyze the magnitude of the most commonly used project performance metrics in campaign's success: project's goal, project's subject, geographical location, duration of a campaign, number of backers and amount funded (N = 1000). Data is obtained from KickStarter.com server. Our findings suggest that participation grows from previous interactions with crowdfunding, other experience is insignificant. Common performance metrics have impact on campaign's success, though our findings propose two of them insignificant, namely duration and location. Given previous findings state duration as important determinant of success and lack of data on estimating *willingness to help*, findings carry implications on estimating success determinants of CF projects.

Аннотация. При планировании краудфандинговой кампании одним из ключевых факторов является так называемая оценка стартапером «*готовности прийти на помощь*» („*willingness to help*") тех людей, которые являются его «*друзьями*» в социальных сетях, иначе говоря его «социальная сеть» („*social network*"). Целью данного исследования является выявление, влияет ли прошлый опыт и вовлеченность в социальные сети на дальнейшее участие предполагаемого инвестора/основателя кампании в краудфандинге. Исследуя различные поведенческие паттерны инвестора/стартапера посредством опроса (отношение к азартным играм, карьерные предпочтения, мотивация участвовать в краудфандинге), мы делаем вывод о том, какую роль играют вышеперечисленные факторы при выборе человека, участвовать или нет в краудфандинговой кампании. В дополнение к данным от опроса (N = 120) мы анализируем наиболее часто используемые показатели эффективности проекта: цели проекта, тема проекта, фактическое географическое местоположение, продолжительность кампании, количество сторонников и объема привлеченных средств (N = 1000). Данные были получены с сервера KickStarter.com.
Наши результаты показывают, что потенциальное участие человека в качестве инвестора/основателя кампании складывается из наличия предыдущего опыта в краудфандинге, другие факторы не так важны. Общие показатели эффективности имеют влияние на успех кампании, хотя наши результаты предлагают незначительную составляющую двух из них, а именно продолжительность кампании и географическое месторасположение. Учитывая то, что в предыдущих исследованиях продолжительность кампании расценивается как важный фактор, определяющий успех, а также отсутствие данных по оценке «*готовности прийти на помощь*», результаты исследования вносят вклад в существующее понимание составляющих успеха краудфандинговых проектов.

Key words: Crowdfunding, social circle, start-up, fundraising, entrepreneurship.

INTRODUCTION

In the last few years a new form of investment has attracted interest of increasing number of initiatives and firms: crowdfunding. In crowdfunding people pool their money together, in order to invest in and support efforts initiated by other people or organizations. It is the system to finance a project or a firm by

* Определяющие факторы успеха краудфандинговых проектов.

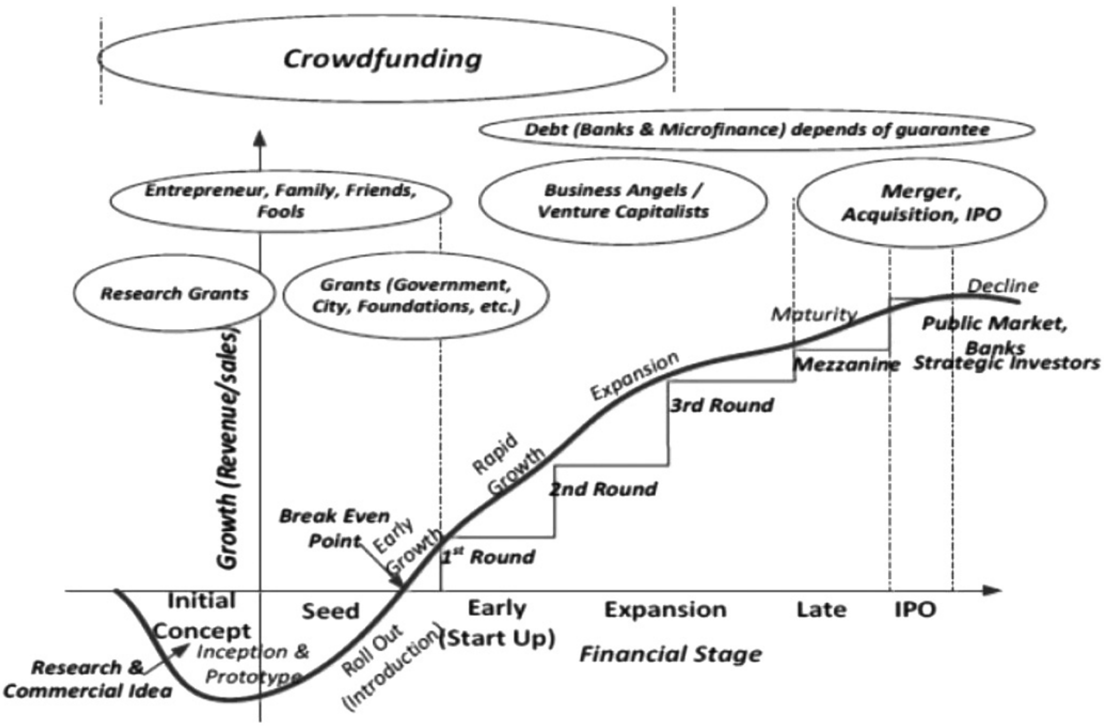

Figure 1.

Source: International Journal of Financial Research.

a group of people instead of professional parties like banks or venture capitalists, allowing individuals to fund entrepreneurs directly even with small amounts. There are about 500 crowdfunding platforms differing in type of fundraising, among them charity, preorder, equity and lending.

This article studies factors influencing the success of crowdfunding campaigns. We attempt to explore differences in investor's utilities from participating in crowdfunding, related to investor's beliefs, behavioral patterns and background. Study provides insights for proper estimation willingness to support entrepreneur's 1-tier social circles, which evidentially play key role in boosting project's economic capital.

Assuming that propensity to engage in social ventures is driven by past experience, we interviewed people on different issues, such as: entrepreneurial experience, career preferences, attitude to traditional sources of finance, gambling experience, motivation to support or create new ventures, etc.

Further, we provide updates on the effect of the most acknowledgeable campaign performance metrics on project's success. A project is considered successful if the declared threshold (i. e. goal) was met by or before the deadline, whereas failed in opposite. Such parameters as project's goal, project's subject and location, duration of a campaign, number of backers and amount raised, stated to influence success, according to the majority of studies. Our findings propose two of them insignificant, namely duration and location.

LITERATURE REVIEW

Crowdfunding seems an ideal match for entrepreneurs seeking for 'societal' value more than for economic gain. While the entrepreneurs act as catalysts, members of the crowd take over various roles at the same time, from co-founders to funders as well as to customers and co-workers. To enable this collaborative opportunity development, the entrepreneurs (1) need to improve their *Culture Capital* (*CC*) in the form of a careful comportment of the languages and values; (2) be prepared to actively work on building *Symbolic Capital* (*SYMC*), for example by emphasizing the societal benefit (legitimacy) of their ideas and by providing respectful interaction with the crowd. When this happens, *Social Capital* (*SC*) is actually transformed into *Economic Capital (EC)* — small monetary contributions of all kinds can sum up to build enough resources to boldly address social needs (Lehner M., 2014).

OPPORTUNITY

There are three main issues to develop "Opportunity": 1) Understand — estimate network support size and their *willingness to participate*; choose appropriate platform; 2) Activate — ask network influencers, ask people likely to support; 3) Expand — connect through structural holes, build reputation.

Many crowdfunding project creators have trouble estimating their network size and who is willing to

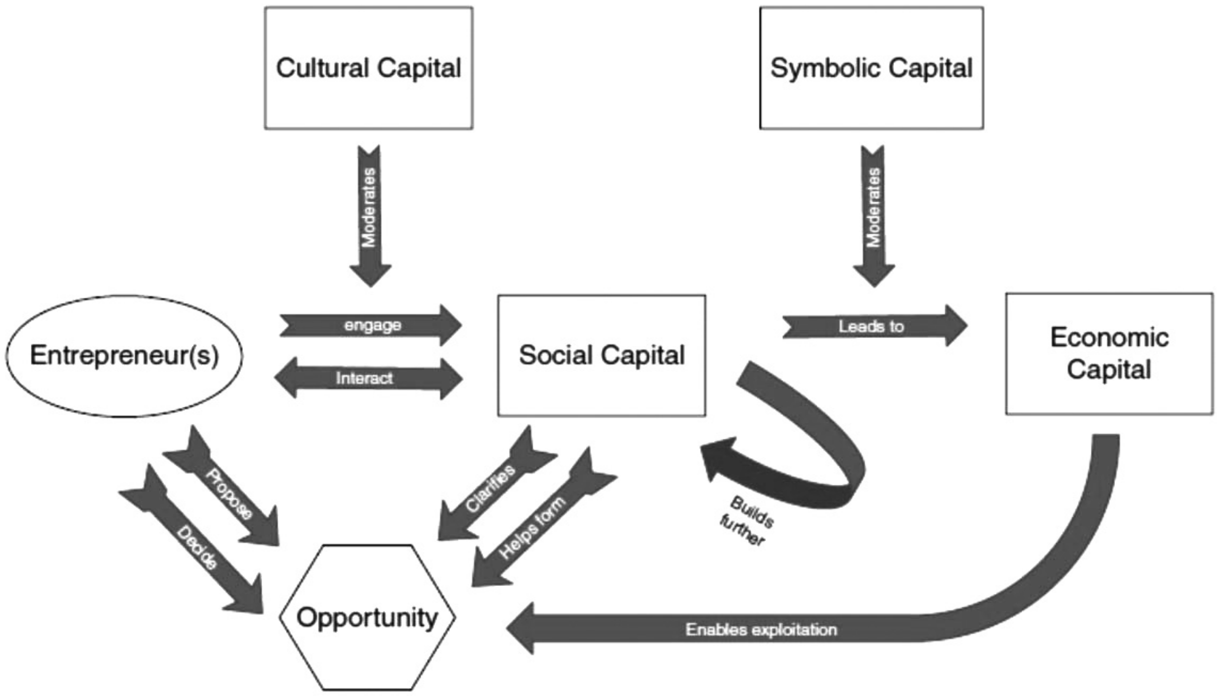

Figure 2.

Source: Entrepreneurship & Regional Development: An International Journal.

give, which is leading them to choose overly ambitious funding goals or spam connections. Entrepreneur should clearly understand general motivations that will drive people to back his future project, choosing platform type accordingly. Donation and reward-based platforms (charity and pre-order) typically are used when the founder aims to get initial capital using emotionally attached fan crowd. Such fans are driven by empathy towards the project. Equity or lending crowdfunding are likely to attract crowd, driven by project's future financial profits.

Declaring a "non-for-profit" status of a project may positively affect the success chances of entrepreneurs to reach their capital targets (Belleflamme, Lambert, Schwienbacher, 2013). Pitcher (2014) provides evidence that non-profit projects are significantly more likely to reach their minimum funding goals. At the same time, however, they have fewer funding givers and obtain lower total funding amounts.

Having appropriate non-financial motivation, heading the project is essential for charity. Building campaign on some basic value, generally accepted by society, may sound like a good strategy. There is evidence from Jian and Shin (2015), showing that belief in freedom of content, altruism, and contributing to communities emerged as the strongest self-reported motivations (highly valued by donors themselves). But, in contrast, fun, family and friends (FF) motivations were the only positive predictors for actual donation levels. Fun appears to be a clear predictor of donation levels, when FF drives only the number of donations, but not the amount. FF is also a less important motivator for returning donors than it is for the first-timers.

Also, people tend to back activities linked with their personal day-to-day problems, such as health-related causes. There are studies, showing that non-profits, reflecting immediate needs or benefits are more likely to succeed (Saxton G., Wang, L.)

When thinking to start a charity project, entrepreneur should account the future price of giving for his campaign (the amount a donor needs to give in order to provide one dollar of the charity's output). While most (but not all) of the authors across the different strands of the literature agree that the price of giving affects charitable giving, the estimated magnitudes vary widely, and many approaches struggle to cleanly identify these effects. For example, a charity that spends a large share of its revenue on fundraising will have a relatively high price of giving — potentially reducing donations — yet those same fundraising expenditures may attract more and larger donations. Price of giving tends to fall into three broad groups: the role of administrative and fundraising costs; the role of tax preferences; and the role of direct subsidies to giving. Meer (2013) states that price of giving affects the likelihood of achieving the funding goal, but not whether the project receives any donors. Thus, a ten percent increase in the price of giving reduces the likelihood of funding by about 3.6 percentage points. Effect of the price of giving is expanding in competitive environment, as donors obtain possibility to compare projects and shift their giving towards more efficient ones.

SOCIAL CAPITAL (SC)

A lot of authors (Mollick, Dresner, Meece) provide evidence that successful campaigns typically start with raising 30% of funds from well-known individuals (friends and family). Colombo, Franzoni, Rossi-Lamastra and Lehner, suggest that in addition to relying on social contacts established *outside* a crowdfunding platform (e. g., family and friends, Facebook or LinkedIn contacts), a project proponent may develop an additional stock of social capital *within* that platform by establishing relationships with other proponents and backers. We call this type of social capital *internal* and hold it separate from the *external* social capital composed of family and friends.

Internal SC appears, due to social *reciprocity* — social contacts within communities may induce community members to finance entrepreneurial initiatives in compliance with *social obligations*, that is, norms of (specific and generalized) reciprocity.

The results indicate that a one-standard deviation increase in *Internal Social Capital* is associated with a predicted increase of 5.1 early backers (from 13.8 to 18.9), which equals a 37.1% increase from the initial value.

Number of backers is crucial for project's success. According to Lichtig (2015), only 24% of projects that fail have more than 25 backers. Conversely, only 17% of successful projects had less than 25 backers. Thus successfully funded projects tend to have many backers and *vice versa*.

CULTURE CAPITAL (CC)

Some authors suggest that geography strongly influences the perception of Culture Capital (CC) by potential backers. Having a local community of artists and creative individuals seems to increase the quality of projects produced by nearby founders. (Mollick, 2014). But, Agrawal, Catalini and Goldfarb suggest that investment patterns over time are not strongly related to the geographic distance between artist and funder after controlling for the artist's offline social network. Different responses relate to the likelihood that friends and family (tier 1 circle, who are disproportionately local) identify a given artist as a worthy recipient of funds. Controlling for preexisting offline social networks, we see little difference between local and distant investment patterns. Nevertheless, cultural differences play important role in project's success. Individuals are more likely to support charities and borrowers in need when they can empathize. It seems that cultural similarity may allow for such empathy. Cultural difference can also rein up desire to back the project. Burtch, Ghose and Wattal are giving evidence in their study of pro-social lending: an increase of one standard deviation in the cultural differences between lender and borrower countries is associated with 30 fewer lending actions, while an increase of one standard deviation in physical distance is associated with 0.23 fewer lending actions.

SYMBOLIC CAPITAL (SYMC)

Even a growing social capital and appropriate magnitude of culture capital (leading to only mutual understanding) cannot guarantee a successful funding. For that to happen, strong *SYMC* is necessary, which acts as a catalyst in the transformation of *SC* into *EC*. There are many types of activities, helping to build *SYMC*. Designing video clips, describing entrepreneur's goal is important. Mollick (2014) shows that having no videos would result in a 15% chance of success, and videos make the chance of success 37%. He also states that increasing goal size as well as duration decreases the chances of success, possibly because longer durations are a sign of lack of confidence. Lichtig (2015) provides evidence that the number of projects a creator has previously launched is highly associated with the number of backers a project will get. Green (2014) designates that choosing perks (rewards) is substantial for donors, since most of successful campaigns offer four or five perks at increasingly higher suggested giving levels. The project owner needs to make sure the affordable perks don't run out too fast, or he risks losing potential backers who can't afford steeper offerings. But, entrepreneur should keep in mind a problem of non-appropriate reward in crowdfunding, which is linked with first-timers syndrome. Lawton and Marom (2010) provide us with example of some intrinsic reward, when backer contributes a small amount of money, but, thanks to his huge network, such a backer gives the project a big crowd of potential investors (through sharing the project he liked with his friends). In fact, that particular investor with no money and big amount of friends made all the work (he catalyzed the major part of funding), but received nothing (or almost nothing) for his efforts. Such a problem can catalyze "first timer's syndrome" — the backer might never come back. That's reason enough to prognosticate that in the future, most successful crowdfunding sites will offer mechanisms to map non-monetary inputs into rewards and not choose a typical strategy, with increasingly larger sizes of investments parlayed into more premium rewards.

Launching the campaign by simultaneously going live on the crowdfunding platform site, announcing the campaign on the organization's social media pages and website, distributing a news release to the entire media list and sending a personalized e-mail to everyone in the organization's distribution list can be essential for building trust (Green, 2014).

Our research makes the following contributions: (1) Due to all previous findings are based on assumption that crowdfunders enjoy the same increase in utility, irrespective of their taste parameter, i.e. each project has an underlying propensity, that propensity varies across projects and among cultural groups in general, but not among individuals. Therefore, we investigate unexplored impact of character reference on utility, which person expects to obtain when contributing to social venture. This impact reflects estimation of social network *willingness to help* and response to translated SYMC, resulting in change of *economic capital* obtained; (2) Our KickStarter. com data analysis suggests no effect of campaign duration on project's success, which was stated in previous findings as influential parameter.

DATA AND METHODOLOGY

KICKSTARTER. COM

Our sample comprises data on more than a thousand projects, loaded from kickstarter. com server, namely, goal, amount pledged, number of backers, project's subject, duration and location.

Our dependent variable, *Project's status*, captures whether project was successful or not. We include four independent variables, serving a base for our model: *project category*, i.e. its subject; *goal*, namely declared amount of funding; *amount pledged* and *number of backers*, who supported the project. Other variables, such as project's *duration* (which is also declared by venturer) and *location* of startup were found insignificant.

OLS regression of the following form was used:

$$Ps = \alpha + \beta_{pi}Pi + \beta_{g}G + \beta_{bk}Bc + \beta_{p}P + \varepsilon$$

where, *Ps* stands for the status of the project (was it successful, failed or canceled); *Pi* reference the project category (Art, food, etc); *G* is amount of money, declared by project's creator (goal); Bc is number of people (backers), supported the project; P stands for total amount of money (amount pledged), resulting the campaign.

All the coefficients are shown to be significant; R-Squared the coefficient of determination representing how close the regression is to its fitted line, is equal to 0.02. Such a low value suggests existence of other parameters influencing funding results. Our findings are in line with previous studies; however, we indicate insignificants of project's *duration* and *location* to its success.

SURVEY

The survey was being conducted via three social networks: Facebook, LinkedIn and Vkontakte. As the result 105 participants took the survey, among them 65 men (61.9%) and 40 women (38.1%) aged from 17 to 60 and older. (Figure 3). The majority of respondents are from 20 to 39 years old which is defined by the choice of the social networks and also by the peculiarity of the crowdfunding as the "new-born" way of investing not spread among people with traditional way of thinking. Interesting that geographical range varies; and answerers who took the survey came from different countries from all over the world: Belarus, Argentina, Australia, Belgium, France, Hungary, Canada, Egypt, Georgia, Germany, India, Israel, Russia, Serbia, Slovakia, South Africa, Syria, the UK, Ukraine and the US.

When asked about whether respondent considers participating in some project using any type of crowdfunding, such as donation, pre-ordering, equity or loan-based crowdfunding 94 (89.5%) answerers said "yes" and only 11 (10.5%) replied negatively. The most accurate question: "Have you ever participated in crowdfunding campaign?" was designed to ask straight and beforehand about whether person participated or not in crowdfunding as backer/creator or he would like to take part in it in future. Surprisingly, but most of the respondents (40%) have already supported the project, only 4 (3.8%) were project owners, 33 respondents (31.4%) didn't deal with crowdfunding, and 26 answerers (24.8%) never participated but "would like to". (Figure 2). All of the

Table 1.

	Estimate	Std. Error	t-value	Pr (>|t|)
(Intercept)	3.430e+00	4.054e-03	846.136	< 2e-16 ***
Pi	−1.165e-07	7.988e-09	−14.582	< 2e-16 ***
G	−1.708e-07	1.151e-08	−14.836	< 2e-16 ***
Bc	8.419e-05	6.658e-06	12.645	< 2e-16 ***
P	5.660e-07	8.057e-08	7.026	2.15e-12 ***

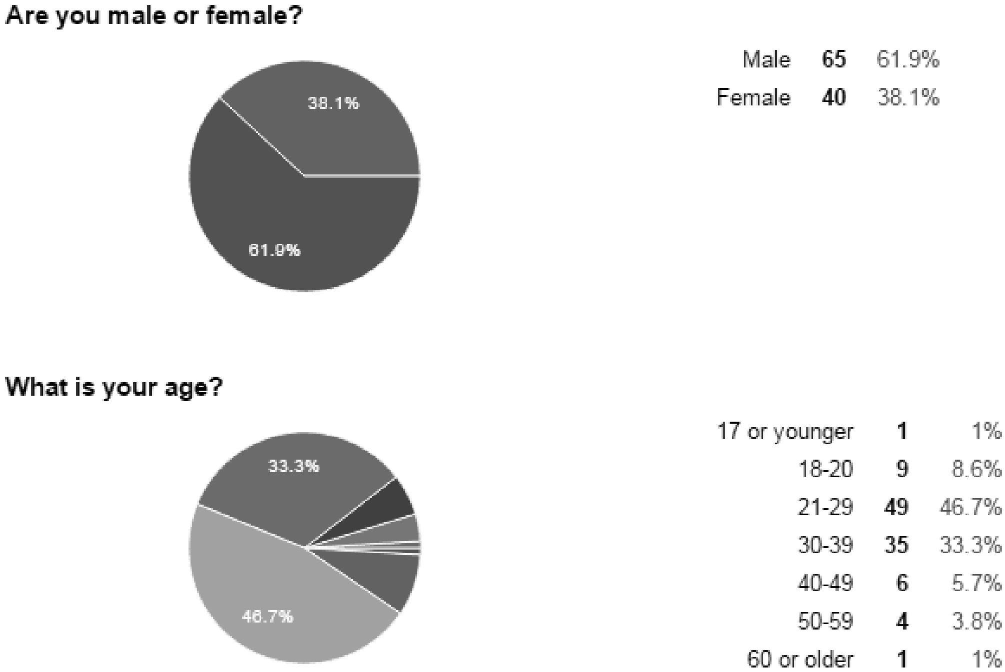

Figure 3.

respondents are familiar with most popular crowdfunding platforms such as Kickstarter, Indiegogo and Rockethub, also the answerers added their national platforms in the field "other" to underline the popularity of crowdfunding as investment in their country. The respondents were asked to assess from 1 to 5 (1 — "not important", 5 — "very important") their motives to participate in crowdfunding as backers: a majority 52.9% reported that empathy and sympathy toward the project is "very important". In addition the answerers (44.1%) reported that it is "not important" to take part in order to strengthen social status; the pursuit to benefit from the project (financial profit) is assessed as "3 medium priority" by 33.3% of people. 34.6% of respondents reported that they would likely back their friend's project.

Also, the respondents were asked to assess from 1 to 5 (1 — "not important", 5 — "very important") their motives to participate in crowdfunding as project creators: 50.5% of the answerers reported that it is "very important" to be motivated financially before starting the campaign, 29.4% replied that it is "very important" to introduce people with your product and expand the awareness of the brand. 33.3% of the respondents are eager to gain approval and potential clients. 30.4% of the participants reported that it is "not important" to learn fundraising skills through the campaign.

Further, we asked questions to learn respondents' opinions on the banking system and also respondents' attitude to gambling, shopping in discount price time. Half of the answerers replied that they had a bank loan and only 15% of this half reported that they had an overdue payment. We didn't ask the aim of the bank loan, but the research shows that the ones, who took the credit, would possibly use the same way of investment in order to start their own business. 78% of the respondents had any type of insurance and 94% of them would use it in future. 65% of applicants replied that they prefer to shop in discount price time, whether 29% don't care when to shop which shows the applicants' wish to get the product with cut price. It is a very important impact on the crowdfunding system: the backers for their donation get the product with discount or pre-order bonus.

Three questions were dedicated to the trials of inventing own business. 36.8% of the applicants reported that they created their own business, 17.9% didn't deal with it and 45.3% answered that they would like to invent their own project. The majority of the business creators found it unsuccessful (24 out of 39 respondents) and most of them used their own savings to start a business. Only 8 people used investors as a start capital. Therefore, we can conclude that answerers are more reliable using their own money rather than taking a loan. Most of the respondents are more eager to work for the profit-making company or their own for-profit business than for other kind of career. (Figure 4).

Finally, we asked a question "Which sum are you ready to donate to the project you like?" The majority of the respondents (53.9%) said that they were ready to donate 10–100 USD, 19 and 18% — 1–10 USD and 100–1000 USD accordingly. Only 4 people can invest

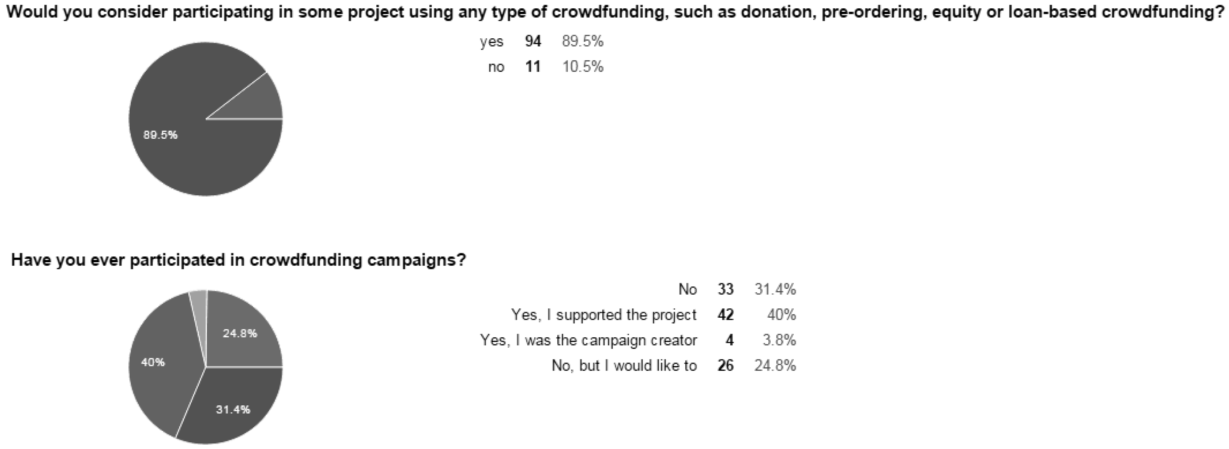

Figure 4.

serious sum of money starting with 1000 USD. And 6 respondents are not ready to invest any money in any crowdfunding projects.

Our dependent variable captures individual's *willingness to participate* in crowdfunding activities. We result including two independent variables, servicing a base for our model: *past experience in crowdfunding* and self-reported *appropriate amount* of donation. Besides, we explored influence of such characteristics as entrepreneurial experience; career preferences; attitude to traditional sources of finance and shopping discounts; gambling experience; motivation to support or create new ventures. None of these variables showed severe effect on willingness to participate and were excluded from model.

OLS regression of the following form is used:

$$Wc = \alpha + \beta_{he}He + \beta_{sr}Sr + \varepsilon$$

where, *Wc* references to the question "Would you consider participating in crowdfunding campaign?"; *He* indicates answers on "Have you ever participated in crowdfunding campaigns?"; *Sr* stands for question "Which sum do you consider appropriate, when investing in crowdfunding activities?"

All the coefficients shown are significant; moreover the R-Squared is equal to 0.24, which is a good value, when dealing with data related to social problems. Past experience in CF gives a positive effect on future participation. Also, increase in considered *appropriate amount* reflects to person's higher trustworthiness in CF, which explains his *willingness to participate*.

DISCUSSION AND CONCLUSIONS

Crowdfunding experienced exponential growth over the last years and can be regarded as an alternative to traditional financiers like banks, private equity funds, venture capital firms or angel investors. Early research provides sufficient amount of recipes, of how to run a successful CF campaign. The most important thing is to transmit a right quality signal to potential investor and to have motivated social network, willing to share signal further.

Our study improves on traditional explanatory models while revealing important insights into the determinants of successful crowdfunding campaign. As stated in previous findings, opportunity recognition is one of the key factors in crowdfunding campaign success. It consists of network support size and their willingness to participate. We assumed that willingness to participate in social ventures is driven by past experience and conducted the survey, interviewing people on different issues. We found no influence of respondent's age, gender, entrepreneurial background, career preferences, attitude to traditional sources of finance, experience in loan-taking, gambling history and motivation to support and create new ventures on decision to participate in crowdfunding activity. Although, we show that previous interactions with crowdfunding increase willingness to participate, i.e. scale up chanc-

Table 2.

	Estimate	Std. Error	t-value	Pr (>|t|)
(Intercept)	0.47910	0.07659	6.255	8.33e-09 ***
He	0.11274	0.02803	4.022	0.000108 ***
Sr	0.09087	0.02982	3.047	0.002909 **

es of positive reaction to transmitted quality signals, which turns in backing the project. Consequently, entrepreneur should estimate opportunity basing on number of social network participants, who already been involved in crowdfunding.

Next, we investigate influence of goal, amount pledged, number of backers, project's subject, duration and location on its final success. As goal, amount pledged, number of backers, project's subject turned to be important success determinants, we state that project's duration and location do not play any significant role in prosperous funding.

We can suggest the following direction for future research: investigation of the effect of project's subject and different types of platforms on subjective expected utility of crowd investor. This issue has been studied by some authors (Hardy, 2013) in the context of price discrimination, but project's subject and platform impacts lacks of research data.

REFERENCES

Agrawal A.K, Catalini C., Goldfarb A. (2011), "The geography of crowdfunding", *National Bureau of Economic Research*, **16820**.

Agrawal A.K, Catalini C., Goldfarb A. (2013), "Some Simple Economics of Crowdfunding", *National Bureau of Economic Research* **14** (1), pp. 63–97.

Ahlers G.K.C. and others, (2012), "Signaling in Equity Crowdfunding", *SSRN Electronic Journal*, p. 49.

Agrawal A., Catalini C., Goldfarb A. (2014), *Crowdfunding: Geography, Social Networks, and the Timing of Investment Decisions,* New York: Wiley.

Bayus B.L., Kuppuswamy V. (2014), "Crowdfunding creative ideas: the dynamics of project backers in Kickstarter", *UNC Kenan-Flagler Research Paper* **2013** (15).

Belleflamme P., Lambert T., Schwienbacher A. (2014), "Individual crowdfunding practices", *Journal of Business Venturing* **29**, pp. 313–333.

Belleflamme P., Lambert T., Schwienbacher A. (2014), "Crowdfunding: Tapping the right crowd". *Journal of Business Venturing* **29** (5), pp. 585–609.

Bradford C.S. (2012), "Crowdfunding and the federal securities law", *Columbia Business Law Review*, p. 150.

Bradford C.S. (2012), "The New Federal Crowdfunding Exemption: Promise Unfulfilled", *Securities Regulation Law Journal* **40** (3), pp. 1–58.

Burtch G., Ghose A., Wattal S., (2013), "Cultural Differences and Geography as Determinants of Online Pro-Social Lending", The MIS Quarterly **38** (3), pp. 773–794.

Colombo M., Franzoni C., Rossi-Lamastra C., (2014), *Internal Social Capital and the Attraction of Early Contributions in Crowdfunding,* Milan: Politecnico.

Colgren B.D. (2014), "The rise of Crowdfunding: Social Media, Big Data, Cloud Technologies", *Technology Practices*, pp. 56–58.

Chen S., Yang C., Lin W., Wei C., (2014), *Will your project get the green light? Predicting the success of crowdfunding campaigns,* Taipei: National Taiwan University.

Danmayr F. (2014), *Archetypes of Crowdfunding Platforms,* Berlin: Springer, p. 117.

Dragojlovic N., Lynd L.D. (2014), "Crowdfunding drug development: the state of play in oncology and rare diseases", *Drug Discovery Today*, pp. 1–6.

Dresner S. (2014), A *Guide to Raising capital on the Internet*, New York: Wiley, p. 272.

Etter V., Grossglauser M., Thiran P. (2013), *Launch hard or go home! Proceedings of the first ACM conference on Online social networks,* Lausanne: Ecole Polytechnique Federale pp. 177–182.

Firth N. (2012), "Crowdfunding successes show value of small donations", *New Scientist* **213**, p. 22.

Gerber E.M., Hui J.S., Kuo P.-Y. (2012), *Crowdfunding: Why people are motivated to post and fund projects on crowdfunding platforms,* Evanston: Northwestern University, p. 10.

Gerber E., Hui J., (2014), "Crowdfunding: Motivations and Deterrents for Participation", *ACM Transactions on Computer-Human Interaction (TOCHI)* **20** (6), Article No. 34.

Green C.H. (2014), *Banker's Guide to New Small Business Finance*, New York: Wiley, p. 224.

Hanley D.F., Bork P. (2012), "Crowdfunding: A New Way to Raise Capital, or a Cut-Back in Investor Protection?", *Insights: The Corporate & Securities Law Advisor*, Volume **26** (6), 44–49.

Hardy W. (2013), "How to perfectly discriminate in a crowd? A theoretical model of crowdfunding", *University of Warsaw Faculty of Economic Sciences Working Papers* **16** (16), 1–28.

Harms M. (2007), "What Drives Motivation to Participate Financially in a Crowdfunding Community?", *SSRN Electronic Journal* **(2269242)**, 80.

Harrison R. (2013), "Crowdfunding and the revitalization of the early stage risk capital market: catalyst or chimera?", *Venture Capital* **15**, 283–287.

Hazen T.L. (2012), "Crowdfunding or Fraudfunding? Social Networks And The Securities Laws — Why The Specially Tailored Exemption Must Be Conditioned On Meaningful Disclosure", *North Carolina Law Review* **90**, 1735–1769.

Hollas J. (2013), "Is Crowdfunding Now a Threat To Traditional Finance?", *Corporate Finance Review* **18**, 27–31.

Hui J., Gerber E., Gergle D., (2014), *Understanding and Leveraging Social Networks for Crowdfunding: Opportunities and Challenges,* Evanston: Segal Design Institute.

Jian L., Shin J., (2014), *Motivations Behind Donors' Contributions to Crowdfunded Journalism,* London: Routledge.

Ibrahim N. (2012), "The Model of Crowdfunding to Support Small and Micro Businesses in Indonesia Through a Web-based Platform", *Procedia Economics and Finance* **4**, 390–397.

Kahneman D., Tversky A. (1979), "Prospect Theory: An Analysis of Decision under Risk", *Econometrica* **27** (2), 263–291.

Karni E. (2005), *Savages' Subjective Expected Utility Model,* Baltimore: Johns Hopkins University, pp. 1–12.

Kim K., Hann I.-H. (2013), "Does Crowdfunding Democratize Access to Capital? A Geographical Analysis", *SSRN Electronic Journal*, 1–35.

Lawton K., Marom D. (2010), *The Crowdfunding Revolution — Social networking meets venture financing,* New York: McGraw Hill, p. 118.

Lehner O.M. (2013), "Crowdfunding social ventures: a model and research agenda", *Venture Capital* **15**, 289–311.

Lehner Othmar M. (2014), "The formation and interplay of social capital in crowdfunded social ventures", *Entrepreneurship & Regional Development: An International Journal*.

Levin J. (1989), "Choice under uncertainty", *European Journal of Operational Research* **41**, 369.

Lichtig B., (2015), "Crowdfunding Success: The Short Story — Analyzing the Mix of Crowdfunded Ventures", *Wharton Research Scholars Journal*.

Lin M., Viswanathan S. (2014), "Home Bias in Online Investments: An Empirical Study of an Online Crowdfunding Market", *Management Science*.

Meer J. (2014), "Effects of the price of charitable giving: Evidence from an online crowdfunding platform", *Journal of Economic Behavior & Organization* **103**, 113–124.

Mitra T., Gilbert E. (2014), *The Language that Gets People to Give: Phrases that Predict Success on Kickstarter,* Atlanta: Georgia Institute of Technology, pp. 49–61.

Mollick E. (2014), "The dynamics of crowdfunding: An exploratory study", *Journal of Business Venturing* **29**, 1–16.

Orelli B. (2012), "Biotech crowdfunding paves way for angels", *Nature biotechnology* **30**, 1020.

Narahari Y. (2008), *Game Theory*, Lecture Notes, pp. 1–13.

Perlstein E.O. (2013), "Anatomy of the Crowd4Discovery crowdfunding campaign", *Springerplus* **2**, 560.

Pitschner S., Pitschner-Finn S., (2014), "Non-profit differentials in crowd-based financing: Evidence from 50,000 campaigns", *Economics Letters* **123,** 391–394.

Saxton G., Wang L., (2011), "The Social Network Effect: The Determinants of Giving through Social Media", *Nonprofit and Voluntary Sector Quarterly,* London: SAGE.

Qiu C. (2013), *Issues in Crowdfunding: theoretical and empirical investigation on Kickstarter,* Rochester: Social Science Electronic Publishing, Inc.

Riedl J. (2013), *Crowdfunding technology innovation,* Los Alamitos: IEEE Computer Society Press, pp. 100–103.

Rubinton B.J. (2011), *Crowdfunding: Disintermediated Investment Banking,* Munich: Munich University, pp. 1–20.

Schwartz A. (2013), "Crowdfunding securities", *Notre Dame Law Review* **88** (13), 1457–1490.

Steinberg S., DeMaria R. (2012), *The Crowdfunding Bible — How to raise money for any startup, video game, or project,* London: ReadMe Publisihing p. 79.

Tomczak A., Brem A. (2013), *A conceptualized investment model of crowdfunding. Venture Capital,* London: Routledge, pp. 1–25.

Voelker T. a., McGlashan R. (2013), *What is Crowdfunding? Bringing the Power of Kickstarter to Your Entrepreneurship Research and Teaching Activities,* Clinton: Small Business Institute, pp. 11–22.

Ward C., Ramachandran V. (2010), *Crowdfunding the next hit: Microfunding online experience goods,* Massachusetts: University of Massachusetts, pp. 1–5.

Wheat R.E. (2013), *Raising money for scientific research through crowdfunding. Trends in Ecology and Evolution,* Boston: Elsevier, pp. 71–72.

Zubrickas R. (2013), *The provision point mechanism with reward money,* Switzerland: University of Zurich pp. 1–20.

Geometry of Economics: Volumetric Distribution Analysis of Economic Continuity and Stability*

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Abstract. While economics derives its value from daily activity of its participants, logically making it a derivative (speed), a traditional approximation of economic variables is carried out using a set of linear and / or nonlinear regression equations and correlation analysis, with no differentiation involved. This explains why the traditional analysis is not capable of identification and prevention of a looming economic crisis: firstly, linear and nonlinear regression value approximation method always relies on continuity assumption of a variable, and secondly, focusing on speed of economics doesn't solve a known limitation of a derivative — its continuity cannot be predicted. This limitation is proposed to be solved with volumetric distribution analysis using volumetric 3D geometry, allowing tracing how distribution of the entire population of the examined variables changes in time and volume as volumetric geometric figures, and what effect it has on continuity of its gradient — the "barometer" of an economic system. Our hypothesis is that a system is stable when it takes a nondegenerate geometric shape and unstable otherwise. An economy can take one shape or another, as volumetric distribution analysis shows, and visualizing it with geometric shapes and respective gradient can help predict its continuity.

Аннотация. В то время как ценность экономики создается за счет повседневной деятельности ее участников, что логически делает ее производной (скоростью), традиционное определение значений экономических переменных осуществляется с помощью набора линейных и/или нелинейных уравнений регрессии и корреляционного анализа без использования дифференциальных уравнений. Это объясняет, почему традиционный анализ не способен к выявлению и предотвращению надвигающегося экономического кризиса: во-первых, метод линейной и нелинейной регрессии всегда опирается на предположение непрерывности переменной, а во-вторых, сосредотачиваясь исключительно на скорости экономики, невозможно решить известное ограничение производной — ее непрерывность не может быть предсказана. Данное ограничение предлагается решить с помощью анализа объемного распределения изучаемых переменных с использованием объемной 3D-геометрии, позволяющей отслеживать изменение распределения совокупности изучаемых переменных во времени и пространстве в виде объемных геометрических фигур, а также влияние, которое она оказывает на постоянство ее градиента — «барометра» стабильности экономической системы. Наша гипотеза заключается в том, что система устойчива, когда она принимает невырожденную геометрическую форму, и нестабильна в обратном случае. Как показывает анализ объемного распределения, экономика может принимать ту или иную форму, и ее визуализация с помощью геометрических фигур и соответствующего градиента поможет предсказать непрерывность ее значений.

Key words: Geometry of economics, volumetric distribution, sustainable economics, economic continuity, income inequality, money paradox.

I.INTRODUCTION

Economics is a mirror of what we do. While economics derives its value from daily activity of its participants, logically making it a derivative (speed), a traditional approximation of economic variables is carried out using a set of linear and/or nonlinear regression equations and correlation analysis, with no differentiation involved. This explains why the traditional analysis is not capable of identification and prevention of a looming economic crisis, because focusing on speed of economics doesn't solve a known limitation of a derivative — its continuity cannot be predicted. This is also true for linear and nonlinear

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regression value approximation — it always relies on continuity assumption of a variable.

Moreover, on the one hand, extrapolation of value derived from a set of its historical values, as the traditional approximation of variables' values method suggests, doesn't take into account a notion of limits that reality may place on a dependent variable. On the other hand, it is known that infinity as a notion can only exist in a theoretical infinite system, but not in a closed system, which is what economic reality is. This mismatch is not only confusing, but it also explains why continuity is assumed in order to justify extrapolation of a dependent variable.

This limitation is proposed to be solved with volumetric distribution analysis using volumetric 3D geometry. We propose visualization of examined variables as volumetric objects, where distribution of one variable is evaluated against another via volumetric visualization over time. Therefore it is possible to trace how distribution of the entire population of the examined variables changes in time and volume as volumetric geometric figures, and what effect it has on continuity of its gradient — the "barometer" of an economic system.

The idea behind volumetric distribution analysis was to develop the methodology of creating accurate models of economic systems, correctly identify principal trends, forecast their future development and help identify actions and points of their application in order to ensure their continuity.

Our hypothesis is that a system is stable when it takes a nondegenerate geometric shape and unstable otherwise. For example, elliptical (2D) or ellipsoid (3D) distribution of variables, approximating multivariate normal distribution of a closed system, represents a more stable/continuous system due to more centered distribution around the mean, while transformation to a hyperbolic (2D) or hyperboloid (3D) distribution would cause instability due to its degenerescence.

In addition, evaluation of the dynamics of change in volume of formed volumetric geometric figures allows us to see not only the transformation of the tested variables in time, but also an onset of critical trends that jeopardize the continuity of the examined variables and the system as a whole.

II. PROBLEM

We will exemplify the problem with two variables — money and number of people. Money as means of exchange reflects economic activity, and stability of the examined system will depend on respective distribution of money between the market participants throughout time.

A traditional analysis of money and number of people distribution is recorded using an exponential distribution function obtained from non-linear regression analysis, where expected value of an exponential random variable X is dependent solely on λ (constant of proportionality, or rate of occurrence):

$$E[X] = 1/\lambda$$

Thus, as λ approaches zero, the value of exponential random variable X approaches infinity.

While an infinite number of people or money is not feasible for practical reasons, we will use the following model with preset boundaries in order to formulate the relationship between the examined variables:

Let's consider the time series:

$$0 < t_1 < t_2 < \ldots < t_{n-1} < T$$

Assume that at the moment of time t_i the wealth value w_i of each market participant (person) is changing due to market activity (e.g. profit or loss) in proportion of the total common wealth (total amount of money) W_i and the probability k_1 to make the profit from the market, where the probability depends on the market participant's current wealth w_i:

$$p_{i+1} = k_1 \cdot W_i$$

The market participant pays taxes to the common wealth at a rate k_2:

$$x_{i+1} = k_2 \cdot p_{i+1}$$

The market participant returns expenses to the common wealth at a rate k_3:

$$e_{i+1} = k_3 \cdot w_i$$

Therefore the total balance of each individual at each time step equals to:

$$w_{i+1} = w_i + p_{i+1} - x_{i+1} - e_{i+1}$$

In an environment where the number of market participants and total common wealth are known, the above model may become unstable at a certain point in time due to the following reasons:

- Common wealth is constant in value, but its distribution over time degenerates following power law distribution rule, stifling future activity (for example, most of the wealth becomes controlled by very few participants who become unwilling to participate in future activity), and

• External factor (s) affecting ability or willingness of the players to participate.

Such a scenario is best visualized when variance of the probability density function approaches zero, while the function itself will approach the delta function:

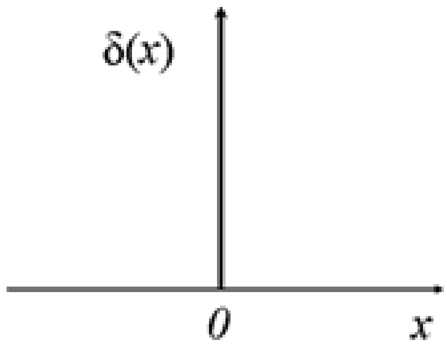

Figure 1.

$$\int_{-\infty}^{+\infty} \delta_{(x)} dx=1 \text{ and } \int_{-\infty}^{+\infty} \delta_{(x-\alpha)} dx=1$$

Therefore, regardless of a certain speed of economy prior to probability density function becoming zero, the system still collapses, but we cannot predict the critical point of such an occurrence using traditional tools and techniques of regression equations and correlation analysis.

III. MODEL

We built a 3D model of market participants and respective money distribution. Each market participant is defined as a unit of volume ($Vp = 1$). Each unit is also assigned a certain value based on the amount of money in possession. Each unit is then located at a level appropriate to its value.

Should such allocation follow the power law distribution, resulting in the majority of people having less money and *vice versa*, we will see a distribution that can be roughly approximated by a volumetric pyramid, with money (value) disproportionately increasing at the top of the pyramid:

As follows from the power law distribution, top one percent of people possess a disproportionately larger amount of common wealth than those at the bottom. Further, about 25% of people possess about 75% of common wealth. The slope of an edge of the triangle/pyramid will get steeper as wealth distribution gets more uneven. While our approach is very simplistic, should the shape of the uneven distribution be hyperbolic, not pyramidal as we have shown above, we can take an average slope of the surface between the peak and the flatter regions and that will still result in a pyramidal or conical shape (shape of the base is not relevant to this discussion).

Conversely, should the wealth distribution between the market participants be more uniform and centered around the median of the population, the resulting shape would be an ellipsoid as shown below.

Our hypothesis is that ellipsoid type distribution makes economic activity of system more stable and thus more continuous than a pyramid or hyperboloid due to a more uniform (normal) distribution. For example, a society, where the middle class possesses the majority of wealth (e.g. U.S. in 1960-s), would show an ellipsoid wealth distribution, making such a society financially stable and generally happy with the state of its economics.

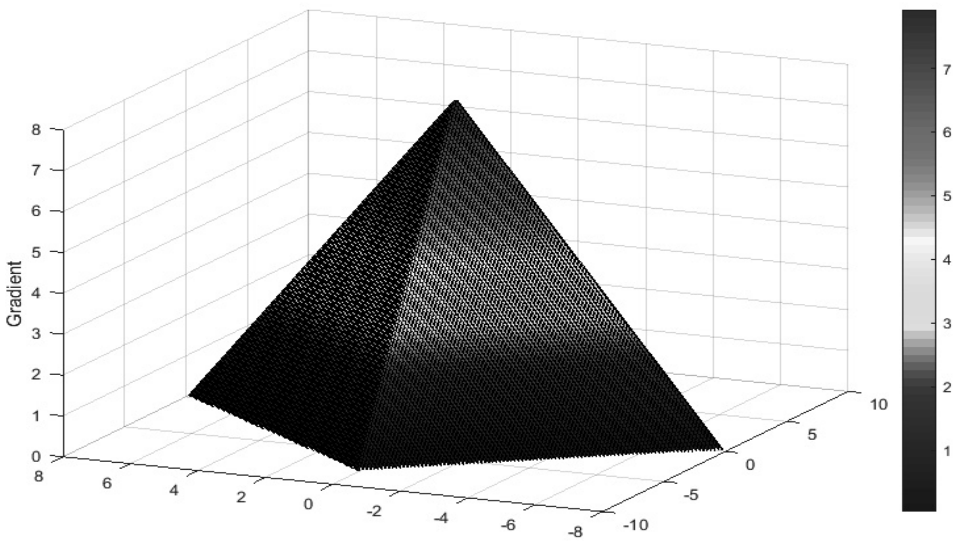

Figure 2.

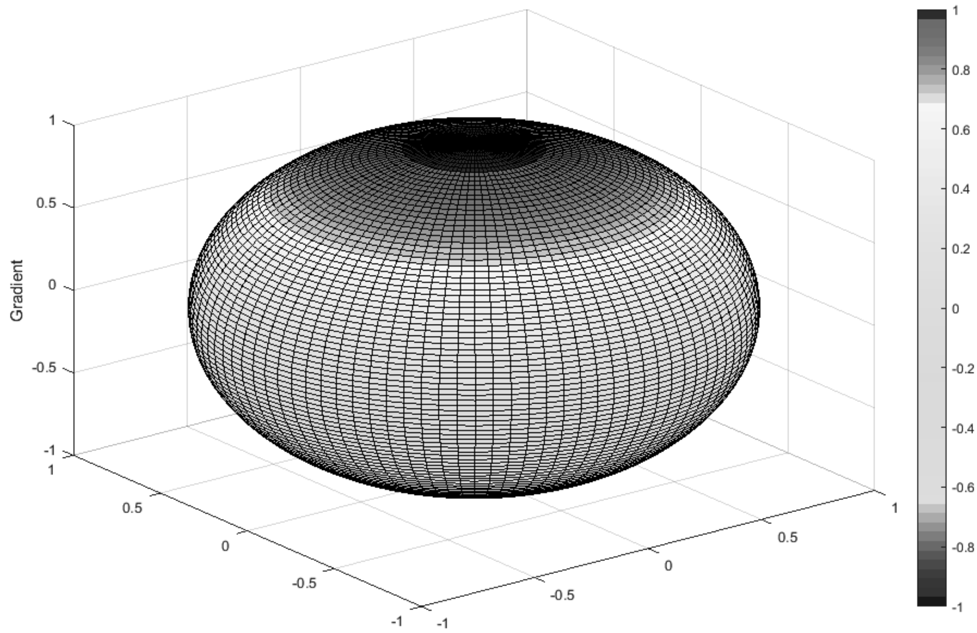

Figure 3.

The shape of wealth distribution figure can be convex or concave. Should the surface of the figure become concave, i.e. with a smaller number of people breaking asymptotically farther away from the mean, it will mean that the economic system as a whole is becoming less stable, thus putting its continuity at risk, and *vice versa* — should the surface of the figure become convex, it will mean that the economic system shows more stability, and thus its continuity will be of less concern.

As a practical example of how we can represent a system geometrically, let's consider the volume of a pyramid with a square base, with height *h* and angle α (angle between one of its non-horizontal edges and the base plane) as follows:

$$V = \frac{2h^3 cot^2(\alpha)}{3}$$

Further, each participant ***n*** occupies a unit of volume. Therefore, if we know the total number of market participants (e.g. ***n*** = 100), then we know the volume of the shape: ***V*** = 100. The number of layers making up the height (levels) of the pyramid h will depend on the disparity of wealth distribution between levels — its height. If there are 10 distinct levels of magnitude, then ***h*** = 10. The angle α is calculated using the following formula, showing interdependence between ***h*** and α:

$$\alpha = \text{arccot}\sqrt{\frac{3V}{2h^3}} = \text{arccot}\sqrt{\frac{3}{20}} = \text{arccot}0.3873 \approx 69°$$

However, if V = 100 (as before) and h = 3, then:

$$\alpha = \text{arccot}\sqrt{\frac{50}{9}} \approx 23°$$

Height ***h*** can also be derived from known values of volume ***V*** and angle α as follows:

$$h = \left(\frac{3V}{2cot^2\alpha}\right)^{1/3}$$

The latter equation will be the link between the vector fields (angle) and scalar fields (height) as matrix calculus equations allow derivation of scalar fields form vector fields, thus turning direction into magnitude. ***ΦA*** scalar field will have some scalar value ***A*** at every point in space, and can be represented by (in 3 dimensions):

$$A = f(x, y, z)$$

Where ***f(x, y, z)*** will demonstrate respective scalar value ***A*** of a unit of volume ***Vp***. The rate of change of this field with respect to time at any point is given by:

$$\frac{dA}{dt} = \frac{dx}{dt} + \frac{dy}{dt} + \frac{dz}{dt}$$

For a vector field, there will be some vector value ***v*** at every point in space, then each component of the vector will be a function of x, y and z independently, so:

$$v = [V1, V2, V3] = [f(x,y,z), g(x,y,z), h(x,y,z)]$$

Then for the rate of change of the vector field at any point, we will have a partial differential equation for each component (3 in this case), for example:

$$\frac{dV1}{dt} = f^{'}(x) + f^{'}(y) + f^{'}(z)$$

By analyzing vector fields and their characteristics, which in turn result in changes to scalar fields and respective divergence of the surface of the figure, we can further analyze how the system changes its shape and if that change adds to its stability and continuity, or not.

The amount of money and its infusion into a financial system is a whole different topic altogether; however, it also has a direct connection to respective distribution for the purpose of continuity of the overall economic system. If one of the variables is artificially altered incorrectly as an attempt to benefit the overall system, then such a system can still run a risk of instability, and vice versa.

For practical reasons, if we equalize energy and activity $\boldsymbol{A}$, we will get that in order to maintain a significant level of activity, or money circulation, we must have a continuous source of money in case of system instability, but the magnitude of activity obtained by infusion of new money into the system may still decrease inversely to profitability $\boldsymbol{P}$ perceived within the system:

$$A = \frac{1}{P}$$

Such inverted relation between business activity and profitability in a closed system could add to an even higher disparity of wealth distribution, making such system even less stable. Tracking distribution of new money with volumetric distribution analysis will help better understand the dynamics and identify stress points early in the game and act accordingly in order to ensure the overall system's continuity.

IV. CONCLUSIONS

An economy can take one shape or another, as volumetric distribution analysis shows, and visualizing it with geometric shapes and gradient can help predict its continuity. Analysis of the vector fields and respective scalar fields, which can be represented as volumetric shapes of the overall distribution of value between examined variables, provides a useful tool for assessment of an economic system's stability.

A continuous, or stable, economy would exist if there was an equal contribution by all participants involved in the economic process — someone creates something and gets something else in exchange. Should there be shortage or excess of contribution or demand, respective distribution and gradient will immediately reflect the distortion, which we will be able to see through respective change of geometric shapes and scalar fields of the examined variables.

ACKNOWLEDGEMENTS

I am grateful to Financial University under the Government of the Russian Federation for their hospitality when this theory was first presented in academic circles. Special thanks to George Kleiner and Alexander Didenko for useful discussions, Roman Libkhen for thoroughly scrutinizing every detail and to Rustam Islamov for implementing the time series equations presented here.

REFERENCES

Kuntsevich, I.V. (2013), *Economical Equilibrium: Geometry of Economics*, Seattle: CreateSpace Independent Publishing Platform.

Richter, M. and Rubinstein, A. (2013), "Back to Fundamentals: Convex Geometry and Economic Equilibrium", *International Journal of Asian Social Science* **3** (10), 2134–2146.

Le. B. (2009), "Classification of Quadrics".

Do, C.B. (2008), "The Multivariate Gaussian Distribution".

Kamer-Ainur, A., Marioara, M. (2007), "Errors and Limitations Associated With Regression And Correlation Analysis", *Statistics and Economic Informatics*, 709.

Danilov, V., Koshevoy, G., Murota K. (2001), „Discrete Convexity and Equilibria in Economies with Indivisible Goods and Money", *Mathematical Social Sciences* **44**, 251–273.

Dragulescu, A.A., Yakovenko, V.M. (2000), „Statistical mechanics of money", *The European Physical Journal* **17**, 723–729.

Manski, C.F. (1999), *Identification Problems in the Social Sciences*, Cambridge: Harvard University Press.